Self Assessment Test -7
Class 10+2
APPLICATION OF DERIVATIVES

1. The radius of a ballon is increase at the rate of 10 cm /sec. At what rate is the surface area of ballon increasing when its radius is 15 cm.
2. Find the pt on the curve $y^2 = 8x$ for which the absicissa and ordinate change at same rate.
3. A man 2 m high walks at a uniform speed of 6 km/h away from a lamp post 6 m high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.
4. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3 cm/sec. Find the rate of the area of the reactangle when length is 12 cm and breadth is 8 cm.
5. An edge of a variable cube is increasing at the rate of 3 cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm long ?
6. A ladder is 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 3cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 3m away from the wall ?
7. Water is leaking from a conical funnel at a uniform rate of $4cm^3/\sec$ through a small hole at the vertex. When the slant height of the water in the funnel is 3cm, find the rate of decrease

of the slant height of water given that semi vertical angle of the cone is 30° .

8. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At the instant when the radius of the circular wave is 8 cm, how fast is the inclosed area increasing ?
9. Show that f(x) = $x^3-6x^2+18x+5$ is an increasing function for all R . Find its value when the rate of increase is least.
10. Prove that f(x) = tan x – 4x is strict decreasing on $\left[\frac{-\pi}{3},\frac{\pi}{3}\right]$.
11. Find the interval in which the f(x) = x^3+5x^2-1 is increasing or decreasing.
12. Find the intervals in which the f(x) = $x^4-8x^3+22x^2-24x+21$ is decreasing or decreasing .
13. Find the intervals of increase and decrease of the f(x) = sin x – cos x , $[0,2\pi]$.
14. Show that f(x) = $\frac{\sin x}{x}$ is strict decreasing on $\left(0,\frac{\pi}{2}\right)$.
15. Find the intervals of increase and decrease of the f(x) = $\frac{\log x}{x}$
16. Prove f(x) = x – [x] is increasing in [0,1).
17. Find the eqn of the tangent line to the curve x = t + sin t , Y = 1 + cos t at t= $\frac{\pi}{4}$.
18. At what pts on the circle $x^2+y^2-2x-4y+1=0$, the tangent is parallel to x-axis
19. Find pts on curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangent is parallel to y-axis.

20. Find the eqn of normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line x + 3y =4.

21. Find the eqn of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line x + 14 y +4 = 0

22. Find the eqn of the normal to the curve $x^2 = 4y$ which passes through the pt (1,2)

23. Find the eqn of tangent to the curve $3x^2 - y^2 = 2$ which are perpendicular to the line x +3 y = 2.

24. Prove that the curve $x = y^2$ and x y = k cut orthogonally if $8k^2 = 1$.

25. Find the eqn of tangent and normal to the curve $y = x^2 + 4x + 1$ at the pt whose abscissa is 3.

26. Find the eqn of all lines having slope -1 that are tangent to the curve y = $\frac{1}{x-1}$.

27. Find the pts on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the pt.

28. Using diff. find an app. Value of

(a) $\sqrt{.037}$ (b) $\sqrt{25.3}$ (c) $(66)^{\frac{1}{3}}$ (d) $(80)^{\frac{1}{4}}$

29. Find an app. Value of $\log_{10}(10.1)$, ,given that $\log_{10} e$ = 0.4343

30. Find an app. Value of sin31° given π = 3.1416

31. If y = $x^3 - 4x$ and x change from 2 to 1.99, find the app. change in the value of y

32. Without using derivative find the maximum and minimum value of (a) f(x) = 3 – 2 sinx (b) f(x) = 2 + x - x^2 (c) f(x) = Isin4x + 3 I

(d) f(x) = sinx cosx (e) f(x) = $\sin^2 x + \cos^4 x$

33. Show that the function f(x) = $x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value .

34. Find the local maxima and minima of the function f(x) = $2x^3 - 21x^2 + 36x - 20$.

35. Find maximum and minimum value of $\sin^4 x + \cos^4 x$.

36. Find two positive numbers whose sum is 24 and whose product is maximum.

37. Show that of all rectangles of given perimeter, the square has largest area.

38. Show that rectangle of maximum perimeter which can be inscribed in a circle of radius 'a' is a square of side $\sqrt{2}a$.

39. Prove that the area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles.

40. A piece of wire 28 m long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible ?

41. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to the diameter of its base.

42. A figure consists of a semi circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

43. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height 'h' is h/3.

44. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum?

45. A window is the form of a rectangle surmounted by a semi circular opening. The total perimeter of the window is 10 m. find the dimension of the window so as to admit maximum light through the whole opening.

46. Show that the semi vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.