

SELF ASSESSMENT TEST -4**CLASS 10+2**

Diff. of infinite series, parametric functions, higher order.

1. If $y = x^{x^{x^{\dots \infty}}}$ find $\frac{dy}{dx}$
2. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$ prove that $\frac{dy}{dx} = \frac{1}{x(2y-1)}$
3. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$ prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$
4. If $y = \sin x^{\sin x^{\sin x^{\dots \infty}}}$ prove $\frac{dy}{dx} = \frac{y^2 \cot x}{1-y \log \sin x}$
5. If $y = e^{x+e^{x+\dots \infty}}$ prove $\frac{dy}{dx} = \frac{y}{1-y}$
6. Find $\frac{dy}{dx}$, $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$
7. Find $\frac{dy}{dx}$, $x = a(\cos \theta + t \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$
8. Find $\frac{dy}{dx}$, $x = a \sin \theta$, $y = a(\cos \theta + \log \tan \theta/2)$
9. Find $\frac{dy}{dx}$ $x = \frac{3at}{1+t^2}$, $y = \frac{3at^2}{1+t^2}$
10. Diff. $x^2 e^{3x}$ w.r.t. $(\log x)^2$
11. Diff. $\log(xe^x)$ w.r.t. $x \log x$
12. Diff. $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ w.r.t. $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$
13. Diff. $\sin^{-1}\left(\frac{2x}{\sqrt{1+x^2}}\right)$ w.r.t. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

14. If $y = Ae^{mx} + Be^{nx}$ prove $\frac{d^2 y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

15. If $y = a \sin(\log x) + b \cos(\log x)$, then prove $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

16. If $x = \tan\left(\frac{1}{a} \log y\right)$, prove $(1+x^2)\frac{d^2 y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

17. If $y = \tan^{-1} x$, prove $(1+x^2)y_2 + 2xy_1 = 0$

18. If $y = (\sin^{-1} x)^2$, prove $(1-x^2)y_2 - xy_1 - 2 = 0$

19. If $y = e^{a \cos^{-1} x}$, prove $(1-x^2)y_2 - xy_1 - a^2 = 0$

20. If $y = \sin(2 \cos^{-1} x)$, prove $(1-x^2)y_2 - xy_1 + 4y = 0$

21. If $y = e^{ax \sin bx}$, prove $y_2 - 2ay_1 + (a^2 + b^2)y = 0$

22. If $x = a(t + \sin t)$, $y = a(1 - \cos t)$ find y_2 at $t = \frac{\pi}{2}$

23. If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$ find y_2 at $t = \frac{\pi}{4}$

24. If $x = 3 \sin t - \sin 3t$, $y = 3 \cos t - \cos 3t$ find y_2 at $t = \frac{\pi}{3}$

25. If $y = \log\left(x + \sqrt{1+x^2}\right)$ prove that $(1+x^2)y_2 + xy_1 = 0$