SELF ASSESSMENT TEST -4
CLASS B.SC-2
SEQUENCES

1. Define convergent , divergent & oscillatory sequences.
2. Define l.u.b. & g.l.b. of a seq.
3. By definition show that $\lim_{n\to\infty} \frac{n^2+1}{2n^2+3} = \frac{1}{2}$
4. By definition,show that the seq $\left\{\log\frac{1}{n}\right\}$ divergent to $-\infty$.
5. If $u_n = 2 + \frac{(-1)^n}{n^2}$,find the least positive integer m s.t. $|u_n - 2| < \frac{1}{10^4} \forall n \ge m$.
6. Let a seq x_n be s.t. $\frac{x_{n+1}}{x_n} \to l$, l>1 then $x_n \to \infty$.
7. Show that $\left\{\frac{n!}{n^n}\right\}$ is a null seq.
8. Give exp to prove that seq $\{a_n + b_n\}$ is convergent but seq $\{a_n\}$ and $\{b_n\}$ are not both convergent
9. Let $\{a_n\}$ be a seq. s.t. $\frac{a_{n+1}}{a_n} \to l$. If $|l| < 1$ then $a_n \to 0$.
10. State & prove cauchy's first thm on limits.
11. State & prove Cauchy's second thm on limits.
12. Show that $\lim_{n\to\infty} \frac{1}{n}\left(\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + ...+ \frac{n+1}{n}\right) = 1$.
13. Show that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.

14. Prove that $\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+.....+\frac{1}{(2n)^2}\right]=0$.

15. Show that $\lim_{n\to\infty}\left(\frac{2}{1}.\frac{3}{2}........\frac{n}{n-1}\right)^{\frac{1}{n}}=1$.

16. Prove that a monotonically increasing seq is cgt iff it is bdd above.

17. Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3

18. Show that the seq $\{a_n\}$ define by $a_1=\sqrt{7}, a_{n+1}=\sqrt{7+a_n}$ cgs to the positive root of the eqn $x^2-x-7=0$.

19. Prove that the seq $\left\{\frac{2n-3}{5n+7}\right\}$ is monotonic increasing, bdd above & cgs to 2/5.

20. Prove that the seq $\{x_n\}$ where $x_n=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...........+\frac{1}{n!}$ is cgt and $2<\lim_{n\to\infty}x_n\le 3$.

21. If $x_n=1+\frac{1}{2}+\frac{1}{3}+..........+\frac{1}{n}-\log n$ then prove that seq is mono decreasing. Also prove that it is cgt.

22. If $a_n>0, b_n>0$, $a_{n+1}=\frac{a_n+b_n}{2}$ *and* $b_{n+1}=\sqrt{a_n b_n}$ then prove that $\{a_n\}$and $\{b_n\}$ are monotonic & both cgs to same limit

23. Prove that seq $\left\{\frac{n^\alpha}{\beta^n}\right\}, \beta>1$ are null seq

24. Prove that the seq $\{a^n\}$ is cgt to zero if 0<a<1

25. Prove that every seq contains a monotone subseq.

26. State & prove Bolzano weirstrass thm.

27. 27.State & prove Cauchy's general principle of convergence.

28. 28.Prove that the seq $\left\{8+\frac{1}{n^3}\right\}$ is a Cauchy seq & find its limit.

29. 29.Apply Cauchy's general principle of cgs to show that
$x_n = 1+\frac{1}{2^2}+\frac{1}{3^2}+.................+\frac{1}{n^2}$ cgs.

30. 30.Prove that following (a) $\left\{\frac{1}{n^3}\right\}$ (b) $\left\{\frac{n}{n+1}\right\}$ seq is Cauchy seq.

31. 31. Using Cauchy's general principle of cgs show that
$x_n = 1+\frac{1}{3}+\frac{1}{5}+..............+\frac{1}{2n-1}$ not cgs

32. 32. State Cauchy stolze thm. Using thm show that $\underset{n\to\infty}{lt}\frac{\log n}{n}=0$