SELF ASSESSMENT TEST -1
CLASS B.A-2

JACOBIANS

1.Define Jacobian of n functions.

2.If $u_1 = \frac{x_1}{x_n},\ u_2 = \frac{x_2}{x_n},\ u_3 = \frac{x_3}{x_n}$ .... $u_{n-1} = \frac{x_{n-1}}{x_n}$ and show that

$$\frac{\partial(u_1, u_2 \cdots\cdots u_{n-1})}{\partial(x_1, x_2 \cdots\cdots x_{n-1})} = \frac{1}{x_n^{\,n+1}} .$$

3.Find $\frac{\partial(y_1, y_2 \cdots\cdots y_n)}{\partial(x_1, x_2 \cdots\cdots x_n)}$, it being given that

$$y_1 = x_1(1-x_2),\ y_2 = x_1 x_2(1-x_3),\ y_3 = x_1 x_2 x_3(1-x_4) \cdots\cdots y_{n-1} = x_1 x_2 \cdots\cdots x_{n-1}(1-x_n) \text{ and } y_n = x_1 x_2 \cdots\cdots x_n$$

4. If

$$y_1 = r(\cos\theta_1),\ y_2 = r\sin\theta_1\cos\theta_2,\ y_3 = r\sin\theta_1\sin\theta_2\cos\theta_3 \cdots\cdots y_{n-1} = r\sin\theta_1\sin\theta_2 \cdots\cdots \sin\theta_{n-2}(\cos\theta_{n-1}) \text{ and } y_n = \sin\theta_1\sin\theta_2 \cdots\cdots \sin\theta_{n-1}$$

prove $\frac{\partial(y_1, y_2 \cdots\cdots y_n)}{\partial(r, \theta_1 \cdots\cdots \theta_{n-1})} = r^{n-1}\sin^{n-2}\theta_1 \sin^{n-3}\theta_2 \cdots\cdots \sin\theta_{n-2}$.

5.Let f(x,y) = (sinx,cosy) and g(x,y)= (x^2, y^2). Let F= fog. Evaluate $J_F(x, y)$

6.prove $J_{f^{-1}}(x, y)$ = x where f(x,y)= $\left(\sqrt{x^2+y^2}, \tan^{-1}\frac{y}{x}\right)$.

7.If u^3 =xyz, $\frac{1}{v} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ and $w^2 = x^2 + y^2 + z^2$, prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -\frac{v(y-z)(z-x)(x-y)(x+y+z)}{3u^2 w(yz+zx+xy)} .$$

8. If u,v,w, are roots of the eqn $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$. prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)} .$$

9.If x + y + z = $u^3+v^3+w^3$, $x^3+y^3+z^3=u^2+v^2+w^2$ and $x^2+y^2+z^2=u+v+w$ prove $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$

10.Show that the functions u = x -2y + z , v = x^2 + 2xy – xz , w = 3x +2y – z are not indep. of one another. Also find the relation between u,v,w.

11.Prove that $ax^2+2hxy+by^2$ and $Ax^2+2Hxy+By^2$ are indep. of each other unless $\frac{a}{A}=\frac{h}{H}=\frac{b}{B}$.

12.Show that the functions u = $x^2+y^2+z^2$ -2zx , v = x + y –z , w = x – y – z are not indep of one another. Also find the relation between them.