SELF ASSESSMENT TEST -3
CLASS B.A-2
RIEMANN INTEGRATION

1. Define partitions, refinement, lower & upper Riemann sums, lower & upper Riemann integral.
2. Let f be a bdd functions define on [a,b] and let P be a partition of [a,b]. If P^* be a refinement of P then $L(P^*, f) \geq L(P, f)$ and $U(P^*, f) \leq U(P, f)$.
3. If a refinement of P contains p pts more than P and |f(x)|< k for all x belongs to [a,b] then $L(P, f) \leq L(P^*, f) \leq L(P, f) + 2pk\lambda$ where λ is the norm of P.
4. Find the upper & lower sums of the f(x) = sin x by dividing the closed interval [0,] into six equal intervals.
5. If f be bdd function define on [a,b] , then $\int_{-a}^{b} fdx \leq \int_{a}^{-b} fdx$
6. If f is a bdd function defined on [a,b],then for every ε >0 , however small $\exists$ δ >0 s.t. $L(P, f) > \int_{-a}^{b} fdx - \varepsilon$ and $U(P, f) < \int_{a}^{-b} fdx + \varepsilon$
7. prove that $\lim_{n\to\infty} \sum_{i=i}^{n} f(\zeta_i)\delta_i$ tends to finite limit iff $\int_{-a}^{b} fdx = \int_{a}^{-b} fdx$.
8. A NASC for a bdd function f to be R- integrable on [a,b] is that to every ε >0 , however small, there exists a partition P s.t. U(P,f) – L(P,f) < ε
9. Prove that every continuous function is R.I.
10. Prove that every monotonic function defined on a closed interval is R.I.
11. If a function f defined on [a,b] is bdd and has a finite number of pts of discontinuity , then f is R.I.

12. Give an exp of bdd function which is not R-integrable over [0,1]

13. Show that the function f define by f(x) = $\begin{cases} 1 & x \quad rational \\ -1 & x \quad irrational \end{cases}$ is not integrable on any interval [0,2].

14. If f(x) = 1-x show that $f \varepsilon \quad R[a,b]$ and $\int_0^1 f dx = \frac{1}{2}$.

15. Let f be the function defined on [0, $\frac{\pi}{4}$] by

f(x) = $\begin{cases} \cos x & x \quad rational \\ \sin x & x \quad irrational \end{cases}$ show that f is not R-integrable

16. Proceeding from the defination compute the integral $\int_a^b x^m dx$

17. Let f be a function defined on [0,1] by f(x) = $\begin{cases} 1 & x \neq \frac{1}{2} \\ 0 & x = \frac{1}{2} \end{cases}$ show that f$\in$ R[0,1] and $\int_0^1 f dx = 1$

18. If f is non negative cont. function on [a,b] & $\int_a^b f dx = 0$, prove that f(x) =0 for all x in [a,b]

19. Let f $\in$ R[a,b] prove that f is integrable on any interval [c,d] $\subset$ [a,b].

20. Let f be the function defined on [0,2] as

f(x) = $\begin{cases} x + x^2 & x \quad rational \\ x^2 + x^3 & x \quad irrational \end{cases}$, evaluate $\int_{-0}^{2} f dx \quad and \quad \int_0^{-2} f dx$