Topics Covered

Limits, Continuity & Differentiability

1. $\lim\limits_{x\to 2} \dfrac{2^x + 2^{3-x} - 6}{2^{-x/2} - 2^{1-x}}$ is equal to

(1) 8 (2) 4

(3) 2 (4) none of these

2. $\lim\limits_{x\to 0} \dfrac{x^3\sqrt{z^3-(z-x)^3}}{\left(\sqrt[3]{8xz-4x^2+\sqrt[3]{8xz}}\right)^4}$ is equal to

(1) $\dfrac{z}{2^{11/3}}$ (2) $\dfrac{1}{2^{23/3}.z}$

(3) $2^{21/3} z$ (4) none of these

3. The value of $\lim\limits_{x\to 0} \dfrac{(1+x)^{1/3}-(1-x)^{1/3}}{x}$ is

(1) $\dfrac{2}{3}$ (2) $\dfrac{1}{3}$

(3) 1 (4) none of these

4. $\lim\limits_{x-y-1} \dfrac{(1+x)(1-x^2)(1+x^2)(1-x^4).....(1-x^{4n})}{\left[(1+x)(1-x^2)(1-x^3)(1-x^4)....(1-x^{2n})\right]^2}$

(1) ${}^{4n}C_{2n}$ (2) ${}^{2n}C_n$

(3) $2.{}^{4n}C_{2n}$ (4) $2.{}^{2n}C_n$

5. The value of $\lim\limits_{x\to 1} \dfrac{x^n + x^{n-1} + x^{n-2} + ...+ x^2 + x - n}{x-1}$ is

(1) $\dfrac{n(n+1)}{2}$ (2) 0

(3) 1 (4) n

6. The value of $\lim\limits_{x\to\infty} \sqrt{x}\left(\sqrt{x+c}-\sqrt{x}\right)$ is

(1) $\dfrac{c}{2}$ (2) $\dfrac{c}{3}$

(3) $\dfrac{c}{4}$ (4) none of these

7. $\lim\limits_{x\to\infty} \dfrac{1-2+3-4+5-6+......-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$ is equal to

(1) $\dfrac{1}{3}$ (2) $-\dfrac{1}{3}$

(3) $-\dfrac{1}{5}$ (4) none of these

8. The value of $\lim\limits_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$ is

(1) $\dfrac{1}{2}$ (2) 1

(3) 0 (4) none of these

9. The value of $\lim\limits_{x\to\infty} \dfrac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}}$ is

(1) $\dfrac{2}{\sqrt{3}}$ (2) $\sqrt{3}$

(3) $\dfrac{1}{\sqrt{3}}$ (4) none of these

10. $\lim\limits_{x\to\infty} \dfrac{n!}{(n+1)!-n!}$ is equal to

(1) 0 (2) ∞

(3) 1 (4) none of these

11. The value of $\lim\limits_{x\to\infty} \dfrac{\sqrt[4]{n^5+2}-\sqrt[3]{n^2+1}}{\sqrt[5]{n^4+2}-\sqrt[2]{n^3+1}}$ is

(1) 1 (2) 0

(3) –1 (4) ∞

12. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4-1}}$ is equal to

(1) 1 (2) –1

(3) 0 (4) none of these

13. $\lim_{x\to\infty} \dfrac{1+\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{3^2}+....+\frac{1}{3^n}}$ is equal to

(1) $\frac{4}{3}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) none of these

14. The value of $\lim_{x\to\infty} \frac{x^5}{5^x}$ is

(1) 1 (2) –1

(3) 0 (4) none of these

15. $\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}\,x}$ is

(1) λ (2) –1

(3) zero (4) does not exist

16. The value of $\lim_{x\to -\infty}\left[\frac{x^4 \sin(1/x)+x^2}{1+|x|^3}\right]$ is

(1) 1 (2) –1

(3) 0 (4) ∞

17. The value of $\lim_{x\to\infty} \frac{x+\cos x}{x+\sin x}$ is

(1) –1 (2) 0

(3) 1 (4) none of these

18. $\lim_{x\to\frac{\pi}{2}} \dfrac{\left(1-\tan\left(\frac{x}{2}\right)\right)(1-x)}{\left(1+\tan\left(\frac{x}{2}\right)\right)(\pi-2x)^3}$ is

(1) $\frac{1}{8}$ (2) 0

(3) $\frac{1}{32}$ (4) ∞

19. $\lim_{x\to 0} \frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^4}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$

is equal to

(1) $\frac{1}{16}$ (2) $-\frac{1}{16}$

(3) $\frac{1}{32}$ (4) $-\frac{1}{32}$

20. The value of $\lim_{x\to 0} \frac{\sqrt[3]{1+\sin x}-\sqrt[3]{2-\sin x}}{x}$ is

(1) $\frac{2}{3}$ (2) $-\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $-\frac{3}{2}$

21. $\lim_{h\to 0} \frac{\ln(1+2h)-2\ln(1+h)}{h^2}$

(1) 1 (2) – 1

(3) 0 (4) none of these

22. The value of $\lim_{n\to\infty} \frac{3^n+2^n}{3^n-2^n}$ *is*

(1) – 1 (2) 1

(3) 0 (4)

23. $\lim_{x\to 0}\left[\frac{in\cos x}{4\sqrt{1+x^2}-1}\right]$ *is equal to*

(1) 2 (2) – 2

(3) 1 (4) – 1

24. If $f(x)=\frac{x^{2n}-1}{x^{2n}+1}$ *then* $\lim_{n\to\infty} f(x)$ *is*

(1) –1 (2) 1

(3) 0 (4) none of these

25. The value of constants a and b so that

$\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=\frac{1}{2}$, *are*:

(1) $a=1,\ b=-\frac{3}{2}$ (2) $a=-1,\ b=\frac{3}{2}$

(3) $a=0, b=0$ (4) $a=2, b=-1$

26. If $\lim_{x\to -\infty}\left(\sqrt{x^2-x+1}-ax-b\right)=0$, then the values of a and b are given by:

(1) $a=-1, b=\frac{1}{2}$ (2) $a=1, b=\frac{1}{2}$

(3) $a=1,\ b=-\frac{1}{2}$ (4) none of these

27. $\lim\limits_{x\to\infty}\dfrac{a_0x^n+a_1x^{n-1}+a_2x^{n-2}+....+a_n}{b_0x^m+b_1x^{m-1}+b_2x^{m-2}+....+b^2}$; m, n > 0, is equal to

(1) 0 (2) ∞

(3) $\dfrac{a_0}{b_0}$ (4) none of these

28. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+2x\_1}{x^2+x+2}\right)^{\frac{6x+1}{3x+2}}$ is equal to

(1) 3 (2) 9

(3) 1 (4) none of these

29. $\lim\limits_{n\to\infty}\dfrac{n^k\sin^2(n!)}{n+2}$, $0<k<1$, is equal to

(1) ∞ (2) 1

(3) 0 (4) none of these

30. $\lim\limits_{x\to0}\left(\dfrac{x^2+5x+3}{x^2+x+2}\right)^x$ equals

(1) e^4 (2) e^2

(3) e^3 (4) e

31. For $x\in R, \lim\limits_{x\to\infty}\left(\dfrac{2x-3}{3x+2}\right)^x$ is equal to

(1) e (2) $-\dfrac{1}{2}$

(3) 0 (4) $\dfrac{1}{2}$

32. $\lim\limits_{x\to a}\left(\dfrac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$, $a\neq n\pi$, n is an integer equals

(1) $e^{\cot a}$ (2) $e^{\tan a}$

(3) $e^{\sin a}$ (4) $e^{\cos a}$

33. $\lim\limits_{x\to0}\dfrac{\tan^{-1}x-\sin^{-1}x}{x^3}$ is equal to

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{2}$

(3) 1 (4) –1

34. The value of $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$ is

(1) $\dfrac{3}{\sqrt{2}}$ (2) $\dfrac{\sqrt{2}}{3}$

(3) $\dfrac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

35. If $\lim\limits_{x\to0}\dfrac{\log(3+x)-\log(3-x)}{x}=k$, then the value of k is

(1) 0 (2) $-\dfrac{1}{3}$

(3) $\dfrac{2}{3}$ (4) $-\dfrac{2}{3}$

36. Let $f(2)=4$ and $f'(2)=4$.

Then $\lim\limits_{x\to2}\dfrac{xf(2)-2f(x)}{x-2}$ is given by

(1) 2 (2) –2

(3) –4 (4) 3

37. Let $f(a)=g(a)=k$ and their nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4,$$

then the value of k is

(1) 4 (2) 2

(3) 1 (4) 0

38. Let $f(x)$ be a twice differentiable function and $f''(0)=5$, then $\lim\limits_{x\to0}\dfrac{3f(x)-4f(3x)+f(9x)}{x^2}$ is equal to

(1) 30 (2) 120

(3) 40 (4) none of these

39. $\lim\limits_{x\to\infty}\dfrac{\log_e[x]}{x}$, where $[x]$ denotes the greatest integer less than or equal to x, is

(1) 1 (2) – 1
(3) 0 (4) does not exist

40. In a circle of radius r, an isosceles triangle ABC is inscribed with AB = AC. If the ΔABC has perimeter $p=2\left[\sqrt{2hr-h^2}+\sqrt{2hr}\right]$ and area $A=h\sqrt{2hr-h^2}$. where h is the alitude from A to BC, then $\lim_{h\to 0^+}\frac{A}{p^3}$ is

(1) 128 r (2) $\frac{1}{128r}$
(3) $\frac{1}{64r}$ (4) none of these

41. $\lim_{x\to 1}\frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

(1) exists and it equals $\sqrt{2}$
(2) exists and it equals $-\sqrt{2}$
(3) does not exit because $(x-1)\to 0$
(4) does not exist because left hand limit is not equal to right hand limit

42. $\lim_{x\to\infty}\sec^{-1}\left(\frac{x}{x+1}\right)$ is equal to

(1) 0 (2) π
(3) $\frac{\pi}{2}$ (4) does not exist

43. $\lim_{x\to 3}([x-3]+[3-x]-x)$, where [.] denotes the greatest integer functin, is equal to

(1) 4 (2) – 4
(3) 0 (4) does not exist

44. $\lim_{x\to e}\frac{\ln x-1}{|x-e|}$ is equal to

(1) $\frac{1}{e}$ (2) $-\frac{1}{e}$
(3) e (4) does not exist

45. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}$ is

(1) $\frac{3}{2}$ (2) 1
(3) –1 (4) none of these

46. The interval where the functin $\log(1+x)$ is continous, is

(1) $(0,\infty)$ (2) $(-\infty,-1)$
(3) $(-1,\infty)$ (4) none of these

47. If $f(x)=\frac{1}{1-x}$, then the points of discontinuity of the function $f[f\{f(x)\}]$ are

(1) $\{0\}$ (2) $\{0,1\}$
(3) $\{1,-1\}$ (4) none of these

48. If $f(x)=\begin{cases}\frac{e^{[x]+|x|}-2}{[x]+|x|-1}, & x\neq 0\\ -1, & x=0\end{cases}$ ([.] denotes the greatest integer function), then

(1) $f(x)$ is continuous at $x = 0$
(2) $\lim_{x\to 0^+}f(x)=-1$
(3) $\lim_{x\to 0^-}f(x)=1$
(4) none of these

49. The point of discontinuity of the function

(1) $n\pi$ (2) $n\pi\pm\frac{\pi}{2}$
(3) $n\pi\pm\frac{\pi}{4}$
(4) continuous every where

50. The function $f(x)=x-[x]$, where $[x]$ denotes the greatest integer function

(1) is continous everywhere
(2) is continous at integral points only
(3) is continuous at non-integral points only
(4) none of these

51. The function $f(x)=(x)$, where (x) denotes the smallest integer $\geq x$, is
(1) continuous everywhere
(2) continuous at integral point only
(3) continuous at non-integral points only
(4) none of these

52. The set of points of discontinuity of the function $f(x)=\log|x|$ is
(1) $\{0\}$ (2) ϕ
(3) $\{1,-1\}$ (4) none of these

53. The set of points of discontinuity of the function $f(x)=|\sin x|$ is
(1) $\{n\pi : n\in I\}$ (2) $\left\{(2n+1)\frac{\pi}{2} : n\in I\right\}$
(3) ϕ (4) none of these

54. The set of points discontinuity of the function $f(x)=\frac{|\sin x|}{\sin x}$ is
(1) $\{0\}$ (2) $\{n\pi : n\in I\}$
(3) ϕ (4) none of these

55. The function $f(x)=(\sin 2x)^{\tan^2 2x}$ is not defined at $x=\pi/4$. The value of $f(\pi/4)$ so that f is continuous at $x=\pi/4$, is
(1) $\sqrt{e}$ (2) $1/\sqrt{e}$
(3) 2 (4) none of these

56. The values of a, b and c which makes the function $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0\end{cases}$

continuous at x = 0, are
(1) $a=-\frac{3}{2}, c=\frac{1}{2}, b=0$ (2) $a=\frac{3}{2}, c=\frac{1}{2}, b\neq 0$
(3) $a=-\frac{3}{2}, c=\frac{1}{2}, b\neq 0$ (4) none of the above

57. If $f(x)$ be a continuous function and $g(x)$ be discontinuous, then
(1) $f(x)+g(x)$ must continuous
(2) $f(x)+g(x)$ must be discontinuous
(3) $f(x)+g(x)$ for all x
(4) can't say

58. The set of points of discontinuity of the function $f(x)=\lim_{x\to\infty}\frac{(2\sin x)^{2n}}{-(2\cos x)^{2n}}$ is given y
(1) R (2) $\left\{n\pi\mp\frac{\pi}{3}, n\in I\right\}$
(3) $\left\{n\pi\pm\frac{\pi}{3}, n\in I\right\}$ (4) none of these

59. If $f(x)$ is continuous in $[0,1]$ and $f\left(\frac{1}{2}\right)=2$, then $\lim_{x\to\infty} f\left(\frac{\sqrt{n}}{2\sqrt{n}+1}\right)$ is equal to
(1) 0 (2) ∞
(3) 2 (4) none of these

60. If the function $f(x)$ defined by $f(x)=\begin{cases}\frac{\log(1+3x)-\log(1-2x)}{x}, & x\neq 0\\ a, & x=0\end{cases}$ is continuous at x = 0, then the value of a is
(1) 5 (2) 1
(3) –1 (4) none of these

61. If $f(x)=\int_0^x t\cos\frac{1}{t}\,dt$, then the number of points of discontinuity of $f(x)$ in the interval $(0,\pi)$ is
(1) 1 (2) 2
(3) 0 (4) none of these

62. If $y=\frac{1}{t^2-t-6}$ and $t=\frac{1}{x-2}$, then the values of x which make the function, y discontinuous, are

(1) $2, \frac{2}{3}, \frac{7}{3}$ (2) $2, \frac{3}{2}, \frac{7}{3}$

(3) $2, \frac{3}{2}, \frac{3}{7}$ (4) none of these

63. If $f(x)=\frac{1}{1-x}$, then the points of discontinuity of the function $f^{3n}(x)$ is / are

where $f^{n} = fof\ ....of\ (n\ times)$, are

(1) $x = 2$ (2) $x=\{0, 1\}$

(3) $x=-1$

(4) continuous everywhere

64. If $f(x)=\frac{1}{x^2-17x+66}$, then $f\left(\frac{2}{x-2}\right)$ is discontinuous at x is equal to

(1) $2, \frac{7}{3}, \frac{25}{11}$ (2) $2, \frac{8}{3}, \frac{24}{11}$

(3) $2, \frac{7}{3}, \frac{24}{11}$ (4) none of these

65 Let $f(x)=[3+2\cos x], x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$,

where $[.]$ denotes the greatest integer function. The number of points of discontinuity of $f(x)$ is

(1) 3 (2) 2

(3) 5 (4) none of these

66. Let $f(x)=\left[x^3-3\right]$, *where* $[.]$ denotes the greatest integer function. Then the number of points in the interval (1, 2) where the function is discountinous, is

(1) 4 (2)2

(3) 6 (4) none of these

67. The Dirichlet function defined as

$$f(x)=\begin{cases}1, if\ x\ is\ rational\\ 0, if\ x\ is\ irrational\end{cases}, is$$

(1) continuous for all real x

(2) continuous only at some values of x

(3) discontinuous for all real x

(4) discontinuous only at some values of x

68. The set of points of continuity of the function

$$f(x)=\sqrt{\frac{1}{2}-\cos^2 x}\ is$$

(1) $\left\{x:\frac{\pi}{4}+2n\pi\le x\le\frac{3\pi}{4}+2n\pi, n\in I\right\}$

(2) $\left\{x:\frac{5\pi}{4}+2n\pi\le x\le\frac{7\pi}{4}+2n\pi, n\in I\right\}$

(3) $\left\{x:\frac{\pi}{4}+2n\pi\le x\le\frac{3\pi}{4}+2n\pi, n\in I\right\}$ $\cup\left\{x:\frac{5\pi}{4}+2n\pi\le x\le\frac{7\pi}{4}+2n\pi\right\}$

. (4) none of these.

69. The function $f(x)=[x]^2-[x]^2$ (*where* $[y]$ is the greatest integer less than or equal to y), is discontinuous at

(1) all integers

(2) all integers except 0 and 1

(3) all integers except 0

(4) all integers except 1.

70. If a function $f(x)$ is defined as

$$f(x)=\begin{cases}-x & x<0\\ x^2 & 0\le x\le 1\\ x^2-x+1 & x>1\end{cases}$$ then

(1) $f(x)$ is differrentiable at x = 0 and x =1

(2) $f(x)$ is differentiable at x = 0 but not at x = 1

(3) $f(x)$ is differentiable at x = 1 but not at x = 0

(4) $f(x)$ is not differentiable at x = 0 and x = 1

71. Let $f(x)=a+b\,l\,x\,l+c\,l\,x\,l^4$, where a, b and c are real constants. Then $f(x)$ is differentiable at x = 0, if

(1) a = 0 (2) b = 0

(3) c = 0 (4) none of these

72. The set of points where the function $f(x)=|x-2|\cos x$ is differentiable is

(1) $(-\infty, \infty)$ (2) $(-\infty, \infty)\backslash(2)$

(3) $(0, \infty)$ (4) none of these

73. If $f(x)=\begin{cases} \frac{x}{1+|x|}, & |x|\ge 1 \\ \frac{x}{1-|x|}, & |x|<1 \end{cases}$ then $f(x)$ is

(1) discontinuous and non-differentiable at $x=-1, 1$ *and* 0

(2) discontinuous and non-differentiable at $x=-1$, where as continuous and differentiable $x = 0$ and $x = 1$

(3) discontinuous as non-differentiable at $x = -1$ and $x = 1$, whereas continuous and differentiable at $x = 0$

(4) none of these

74. Let $f: R \to R$ be a differentiable function and $f(1)=4$. Then the value of $\lim_{x\to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

(1) $8f'(1)$ (2) $4f'(1)$

(3) $2f'(1)$ (4) $f'(1)$

75. The left hand derivative of $f(x)=[x]\sin(\pi x)$ at $x = k$, an integer and $[x]$ = greatest integer $\le x$ is

(1) $(-1)^k (k-1)\pi$ (2) $(-1)^{k-1}.(k-1)\pi$

(3) $(-1)^k . k\pi$ (4) $(-1)^{k-1}.k\pi$

76. If $f(x)=\frac{x}{1+x}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+.....\infty$ then at $x=0$, $f(x)$

(1) has no limit (2) is discontinuous

(3) is continuous but not differentiable

(4) is differentiable

77. If $f(x)=\sqrt{1-\sqrt{1-x^2}}$, then at $x=0$

(1) $f(x)$ is differentiable as well as continuous

(2) $f(x)$ is differentiable but not continuous

(3) $f(x)$ is continuous but not differentiable

(4) $f(x)$ is neither continuous not differentiable

78. If $f(x)$ $f(x)=\sqrt{1-e^{-x^2}}$, then at $x=0$ $f(x)$ is

(1) differentiable as well as continuous

(2) continuous but not differentiable

(3) differentiable but not continuous

(4) neither differentiable nor continuous

79. If $f(x)=\begin{cases} x[x], & 0\le x<2 \\ (x-1)[x], & 2\le x\le 3 \end{cases}$, where [.] denotes the greatest integer function, then

(1) both $f'(1)$ *and* $f'(2)$ do not exist

(2) $f'(1)$ exists but $f'(2)$ does not exist

(3) $f'(2)$ exists but $f'(1)$ does not exist

(4) both $f'(1)$ and $f'(2)$ exists

80. If $f(x)=\begin{cases} 3, & x<0 \\ 2x+1, & x\ge 0 \end{cases}$, then

(1) both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$

(2) $f(x)$ is differentiable but $f(|x|)$ is not differentiale at $x = 0$

(3) $f(|x|)$ is differentialbe but $f(x)$ is not differentiable at $x = 0$

(4) both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$

81. If $f(x)=\begin{cases} 4, & -3<x<-1 \\ 5+x, & -1\le x<0 \\ 5-x, & 0\le x<2 \\ x^2+x-3 & 2\le x<3 \end{cases}$, then $f(|x|)$ is

(1) differentiable but not continuous in (–3, 3)

(2) continuous but not differentiable in (–3, 3)

(3) continuous as well as differentiable in (–3, 3)
(4) neither continuous nor differentiable in (–3, 3)

82. $f + g$ may be a continuous function if
(1) f is continuous and g is discontinuous
(2) f is discontinuous and g is continuous
(3) f and g both are distcontunous
(4) none of the above

83. If $f(x)=2x+|x-x^2|, -1 \le x \le 1$, then $f(x)$ is
(1) continuous but not differentiable in [–1, 1]
(2) continuous as well as differentbaile in [–1,1]
(3) differentiable but not continuous in [–1, 1]
(4) neither differentiable nor continuous in[–1, 1]

84. Let [x] denotes the greatest integer less than or equal to x. If $f(x)$ $f(x)=[x \sin \pi x]$ then $f(x)$ is
(1) not continuous at $x = 0$
(2) not continuous is (–1, 1)
(3) not differentiable at $x = 1$
(4) differentiable in (–1, 1)

85. The function $f(x)=\dfrac{x}{1+|x|}$ is
(1) continuous for all x but not differentiable at $x = 0$
(2) continuous as well as differentiable for all x
(3) neither continuous nor differentiable at $x = 0$
(4) differentiable for all x but not continuous at $x = 0$

86. The function $f(x)=x-|x-x^2|$ $-1\le x \le 1$ is continuous on the interval
(1) $[-1, 1]$ (2) $(-1, 1)$
(3) $[-1, 1] \setminus \{0\}$ (4) $(-1, 1) \setminus \{0\}$

87. If f is an even function such that $\lim_{h \to 0} \dfrac{f(h)-f(0)}{h}$ has some finite non-zero value, then
(1) f is continuous and derivable at $x = 0$
(2) f is continuous but not derivable at $x = 0$
(3) f may be discontinuous at $x = 0$
(4) none of the above

88. Let $f(x+y)=f(x).f(y)$ for all x, y where $f(0)\ne 0$. If $f(5)=2$ and $f'(0)=3$, then $f'(5)$ is equal to
(1) 6 (2) 0
(3) 1 (4) none of these

89. If $f(x)=\log_{10} x$, then at $x = 1$
(1) $f(x)$ is continuous as well as differentiable
(2) $f(x)$ is continuous but not differentiable
(3) $f(x)$ is differentiable but not continuous
(4) $f(x)$ is neither continuous nor differentiable

90. Let $f(x)=\cos x$ and $g(x)=[x+2]$, where [.] denotes the greatest integer function. The value of $(gof)'\left(\dfrac{\pi}{2}\right)$ is
(1) 1 (2) 0
(3) –1 (4) does not exist

91. Let $f(x)=|x|$ and $g(x)=[x]$, where [.] denotes the greatest integer function. Then $(fog)'(-2)$ is
(1) 0 (2) does not exist
(3) –1 (4) 1

92. Let $f(x)$ be a functiondefined $f(x)=\sin(\pi[x-\pi])$, where [.] denotes greatest integer function, then $f(x)$ is
(1) continuous as well as differentiable
(2) continuous but not differentiable
(3) differentiable but not continuous
(4) none of the above

93. If $f(x)=\sum_{k=0}^{n} a_k\,|x-1|^k$, where $a_1 \in R$, then
(1) $f(x)$ is continuous at $x = 1$ for all $a_k \in R$
(2) $f(x)$ is differentiable at $x = 1$ for all $a_k \in R$
(3) $f(x)$ is differentiable at $x = 1$, provided $a_{2k+1}=0$

(4) $f(x)$ is continuous at $x = 1$, provided $a_{2k} = 0$

94. The values of constants a and b so as to make the function $f(x)=\begin{cases} \frac{1}{|x|}, & |x|\geq 1 \\ ax^2+b, & |x|<1 \end{cases}$, continuous as well as differentiable for all x, are

(1) $a=-\frac{1}{2}, b=\frac{3}{2}$

(2) $a=\frac{1}{2}, b=\frac{3}{2}$

(3) $a=\frac{1}{2}, b=-\frac{3}{2}$

(4) none of these

95. Let $f(x)$ be a continuous function defined for $1\leq x\leq 3$. If $f(x)$ takes rational values for all x and $f(2)=10$, then $f(1.5)$ is equal to

(1) 0 (2) 10

(3) not defined (4) any constant

96. Let $f(x)$ and g (x) be defined by $f(x)=[x]$ and $g(x)=\begin{cases} 0, & x\in \text{integer} \\ x^2, & \text{otherwise} \end{cases}$, where $[x]$ denotesthe greatest integer less than or equal to x. then

(1) gof is continuous for all $x\in R$

(2) gof is differentiable for all $x\in R$

(3) fog is continuous for all $x\in R$

(4) fog is differentiable at all $x\in R$

97. Let $f: R\to R$ be any function. Difine $g: R\to R$ by $g(x)=|f(x)|$ for all x. then g is

(1) onto if f is onto

(2) one-one if f is one-one

(3) continuous if f is continuous

(4) differentiable if f is differentiable

98. The function

$f(x)=\max.\{(1-x),(1=x),2\}, x\in(-\infty,\infty)$, is

(1) contionuous at all points

(2) differentiable at all points

(3) differentiable at all points except at x = 1 and $x = -1$

(4) contionous at all points except at x = 1 and $x = -1$, where it is discontionous.s

1. Which of the following functions is differentiable at $x = 0$?

(1) $\cos(|x|)+|x|$ (2) $\cos(|x|)-|x|$

(3) $\sin(|x|)+|x|$ (4) $\sin(|x|)-|x|$

2. The set of all points where the function $f(x)=\sqrt[3]{x^2|x|}$ is differentiable, is

(1) $[0, \infty)$ (2) $(0, \infty)$

(3) $(-\infty, \infty)$ (4) $(-\infty, 0)\cup(0, \infty)$

3. Let $f(x)=|x|+|\sin x|$, $x\in(-\pi/2, 3\pi/2)$. Then, f is

(1) everhwhere continuous

(2) continuous and differentiable

(3) nowwhere differentiable

(4) none of these

4. If $f(x)=\begin{cases} \frac{\sin\{\cos x\}}{x-\pi/2}, & x\neq\frac{\pi}{2} \\ 1, & x=\pi/2 \end{cases}$, where {.} represents the fractional part function, then $f(x)$ is

(1) continuous at $x=\pi/2$

(2) $\lim_{x\to\pi/2} f(x)$ exists, but $f(x)$ is not continuous at $x=\pi/2$

(3) $\lim_{x\to\pi/2} f(x)$ does not exist

(4) $\lim_{x\to\pi/2} f(x)=-1$

5. If α, β $(\alpha>\beta)$ are the points of discontinuity of the function $f(f(x))$, where

$f(x)=\frac{1}{1-x}$, then the set of values of a for which the points (α, β) and (a, a^2) lie on the same side of the line $x+2y-3=0$, is

(1) (–3/2, 1) (2) [–3/2, 1]

(3) $[1, \infty)$ (4) $(-\infty, 3/2]$

6. The function $f(x)$ given by $f(x)=are\sin\left(\frac{2x}{1+x^2}\right)$ is

(1) every where differentiable such that $f'(x)=\frac{2}{1+x^2}$

(2) such that $f'(x)=\begin{cases}\frac{2}{1+x^2}, & -1<x<1\\ \frac{-2}{1+x^2}, & |x|>1\end{cases}$

(3) such that $f'(x)=\begin{cases}\frac{-2}{1+x^2}, & -1<x<1\\ \frac{+2}{1+x^2}, & |x|>1\end{cases}$

(4) every where continuous

7. The function $f(x)=\cos^{-1}(\cos x)$ is

(1) every where continuous

(2) everh where differentiable such that $f'(x)=1$

(3) not differentiable at $x=n\pi,\ n\in Z$ and $f(x)=1, x\neq n^p$

(4) none of the above

8. The $f(x)=\tan^{-1}(\tan x)$ is

(1) every where continuous

(2) discontinuous at $x=\frac{n\pi}{2}, n\in 1$

(3) not differentiable at $x=(2n+1)\frac{\pi}{2}, n\in 1$ such that $f(x)=1\ for\ all\ x\in R-\left\{(2n+1)\frac{\pi}{2}, n\in 1\right\}$

(4) evere where continuous and differentiable such that $f(x)=1\ for\ all\ x\in R$.

9. The number of points in (1, 3), where $f(x)=a^{[x^2]}, a>1$ *is* not differentiable is

(1) 0 (2) 3

(3) 5 (4) 7

10. Let $f(x)=[|x|], where [.]$ denotes the greatest integer function, then $f(-1)$ is

(1) 0 (2) 1

(3) non - existent (4) none of these

11. Let $f(x)=Degree\ of\ \left(u^{x^2}+u^2+2u+3\right)$.

then at $x=\sqrt{2}$, $f(x)$ is

(1) continuous but not differentiable

(2) differentiable

(3) discontinuous

(4) none of these

12. If $f(x)=\text{sgn}(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ and $g(x)=f(f(x))$, then at $x=0$ $g(x)$ is

(1) continuous and differentiable

(2) continuous but not differentiable

(3) differentiable but not continuous

(4) neither continuous nor differentiable

13. Let $f:[0,1]\to[0,1]$ be a continuous function. Then

(1) $f(x)=x$ for at least one $x\in(0,1)$

(2) $f(x)$ will be differentiable in $[0,1]$

(3) $f(x)+x=0$ for at least one x such that $0\le x\le 1$

(4) none of these

14. If $f(x)=x+\tan x \; and \; g(x)$ is the inverse of $f(x) \; then \; g`(x)$ is equal to

(1) $\dfrac{1}{1+\left[g(x)-x\right]^2}$ (2) $\dfrac{1}{2+\left[g(x)+x\right]^2}$

(3) $\dfrac{1}{2+\left[g(x)-x\right]^2}$ (4) none of these

15. Let $f:R\to R$ be a function given by $f(x+y)=f(x)f(y)$ for all $x,y\in R. If \; f`(0)=2, then \; f(x)$ is equal to

(1) Ae^{x} (2) Ae^{2x}

(3) $2x$ (4) none of these

1. The value of $\lim\limits_{x\to 0}\dfrac{1}{x}\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ is

(1) 2 (2) ∞

(3) does not exist (4) none of these

2. $\lim\limits_{x\to -\infty}\dfrac{x^5\tan\left(\dfrac{1}{\pi x^2}\right)+3|x|^2+7}{|x|^3+7|x|+8}$ is equal to

(1) $-\dfrac{1}{\pi}$ (2) 0

(3) ∞ (4) does not exist

3. $\lim\limits_{x\to 1}\sin||\,|x|-2\,|-3|$ is

(1) sin 2 (2) sin 1

(3) 0 (4) does not exist

4. $\lim\limits_{x\to a}\left\{\left[\left(\dfrac{a^{1/2}+x^{1/2}}{a^{1/4}-x^{1/4}}\right)^{-1}-\dfrac{2(ax)^{1/4}}{x^{3/4}-a^{1/4}x^{1/2}+a^{1/2}x^{1/4}-a^{1/4}}\right]^{-1}-\sqrt{2}^{\log 4a}\right\}$ is

(1) a (2) $a^{3/4}$

(3) a^2 (4) none of these

5. The value of $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos x^2}}{1-\cos x}$ is

(1) $\dfrac{1}{2}$ (2) 2

(3) $\sqrt{2}$ (4) none of these

6. $\lim\limits_{x\to 2}\dfrac{\sqrt{x-2}+\sqrt{x}-\sqrt{2}}{\sqrt{x^2-4}}$ is equal to

(1) $\dfrac{1}{2}$ (2) 1

(3) 2 (4) none of these

7. $\lim\limits_{h\to 0}\left\{\dfrac{1}{h.\sqrt[3]{8+h}}-\dfrac{1}{2h}\right\}$ is equal to

(1) $\dfrac{1}{2}$ (2) $-\dfrac{4}{3}$

(3) $-\dfrac{16}{3}$ (4) $-\dfrac{1}{48}$

8. The value of $\lim\limits_{x\to 2a}\dfrac{\sqrt{x-2a}+\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$ is

(1) $\dfrac{1}{\sqrt{a}}$ (2) $\dfrac{1}{2\sqrt{a}}$

(3) $\dfrac{\sqrt{a}}{2}$ (4) $2\sqrt{a}$

9. $\lim\limits_{x\to -\pi}\dfrac{|x+\pi|}{\sin x}$

(1) is equal to –1 (2) is equal to 1

(3) is equal to π (4) does not exist

10. $\lim\limits_{x\to 0}\left\{\dfrac{\log_e(1+x)}{x^2}+\dfrac{x-1}{x}\right\}$ is equal to

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{2}$

(3) 1 (4) none of these

11. $\lim\limits_{x\to 1}\dfrac{\sum\limits_{r=1}^{n}x^r-1^r}{x-1}$ is equal to

(1) $\frac{n}{2}$ (2) $\frac{n(n+1)}{2}$

(3) 1 (4) none of these

12. The value of $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$ is

(1) $\frac{2}{5}$ (2) $\frac{3}{5}$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}$

13. The value of

$\lim_{x\to\infty} x\left[tab^{-1}\left(\frac{x+1}{x+2}\right)-\tan^{-1}\left(\frac{x}{x+2}\right)\right]$ is

(1) 1 (2) –1

(3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

14. The values of constants a and b so that

$\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=0$

(1) $a=0, b=0$ (2) $a=1, b=-1$

(3) $a=-1, b=1$ (4) $a=2, b=-1$

15. Let $f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

The value of $\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$ is

(1) $\frac{50}{3}$ (2) $\frac{22}{3}$

(3) 13 (4) none of these

16. If $\lim_{x\to 0}\frac{4+\sin 2x+A\sin x+B\cos x}{x^2}$ exists, then the value of A and B are

(1) –2 and –4 (2) –4 and –2

(3) –3 and –2 (4) none of these

17. The graph of the function $y=f(x)$ has a unique tangent at the point $(a,0)$ through which the graph passes. Then

$\lim_{x\to a}\frac{\log_e\{1+6f(x)\}}{3f(x)}$ is

(1) 1 (2) 0

(3) 2 (4) none of these

18. Let $f(x)=x^2-1, 0<x<2$ and $2x+3, 2\le x<3$ The quadratic equation whose roots are $\lim_{x\to 2-0} f(x)$, and $\lim_{x\to 2+0} f(x)$ is

(1) $x^2-6x+9=0$ (2) $x^2-10x+21=0$

(3) $x^2-14x+49=0$ (4) none of these

19. $\lim_{x\to\infty}\frac{a^n+b^n}{a^n-b^n}$, where $a>b>1$, is equal to

(1) –1 (2) 1

(3) 0 (4) none of these

20. The limiting value of $(\cos x)^{1/\sin x}$ as $x\to 0$ is

(1) 1 (2) e

(3) 0 (4) none of these

21. The value of $\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{x^2}$ is

(1) 1 (2) e^{-1}

(3) e^{-2} (4) e^{-3}

22. Let

$$f(x)=\begin{cases} x^2, & when\ x\ is\ an\ integer \\ \frac{k(x^2-4)}{2-x}, & otherwise \end{cases}$$

then $\lim_{x\to 2} f(x)$

(1) exists only when k = 1

(2) exists for every real k

(3) exists for every real k except k = 1

(4) does not exist

23. If $A_i=\frac{x-a_i}{|x-a_i|}, i=1, 2, 3......n$ and $a_1<a_2<a_3.....a_n$, then

$\lim_{x\to a_m}(A_1 A_2 ....A_n), 1\le m\le n$

(1) is equal to $(-1)^m$ (2) is equal to $(-1)^{m+1}$

(3) is equal to $(-1)^{m-1}$

(4) does not exist

24. Let $f(x)=x(-1)^{[1/x]}, x\neq 0,$ where [x] denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0} f(x)$
(1) doesn't exist (2) is equal to 2
(3) is equal to 0 (4) is equal to –1

25. If x > 0 and g is bounded function, then $\lim\limits_{x\to\infty} \frac{f(x)e^{nx}+g(x)}{e^{nx}+1}$ is
(1) 0 (2) $f(x)$
(3) $g(x)$ (4) none of these

26. If $f(x)$ is a polynomial satisfying
$$f(x)f\left(\frac{1}{x}\right)+f(x)+f\left(\frac{1}{x}\right) \text{ and } f(2)>1,$$
then $\lim\limits_{x\to 1} f(x)$ is
(1) 2 (2) 1
(3) –1 (4) none of these

27. The value of $\lim\limits_{x\to 1} f\left[\sin^{-1} x\right]$ is
(1) does not exist (2) 1
(3) 0 (4) $\frac{\pi}{2}$

28. The value of $\lim\limits_{x\to -\infty}\left[\tan^{-1} x\right]$ is
(1) –2 (2) –1
(3) ∞ (4) none of these

29. The vale of $\lim\limits_{x\to l}\left[\sin \sin^{-1} x\right]$ is
(1) 0 (2) does not exist
(3) $\frac{\pi}{2}$ (4) none of these

30. The value of $\lim\limits_{x\to \pi/2}\left[\sin^{-1}\sin x\right]$ is
(1) 1 (2) $\frac{\pi}{2}$
(3) 0 (4) none of these

31. The value of $\lim\limits_{x\to 0^+}\left[\frac{\sin x}{x}\right]$ is
(1) 1 (2) 0
(3) does not exist (4) none of these

32. The value of $\lim\limits_{x\to 0^-}\left[\frac{\sin x}{x}\right]$ us
(1) 1 (2) 0
(3) does not exist (4) none of these

33. The value of $\lim\limits_{x\to 0}\left[\frac{\sin x}{x}\right]$
(1) 0 (2) 1
(3) does not exist (4) none of these

34. $\lim\limits_{x\to 3} \frac{[x]^2-9}{x^2-9}$ is
(1) ∞ (2) 0
(3) does not exist (4) none of these

35. $\lim\limits_{x\to 3^-} \prod\limits_{r=3}^{n} \frac{r^3-8}{r^2+8}$ is
(1) $\frac{2}{7}$ (2) $\frac{7}{2}$
(3) 1 (4) does not exist

36. $\lim\limits_{x\to 0} |x|^{[\cos x]}$ is
(1) 1 (2) does not exist
(3) 0 (4) none of these

37. If $a_1=1$ and $a_{n+1}=\frac{4+3a_n}{3+2a_n}, n\geq 1$ and if a_n has a limit l as $n\to\infty$, then
(1) $l=-\sqrt{2}$ (2) $l=\sqrt{2}$
(3) $l=2$ (4) none of these

38. If [.] denotes the greatest integer function then $\lim\limits_{x\to\infty} \frac{[x]+[2x]+......+[n\pi]}{n^2}$ is
(1) 0 (2) x
(3) $\frac{x}{2}$ (4) $\frac{x^2}{2}$

39. $\lim_{x\to 1+0} \frac{\int_1^x |t-1|\, dt}{\sin(x-1)}$ is equla to

(1) 0 (2) 1

(3) –1 (4) none of these

40. Exhaustive set of points where $f(x)=x|x|$ is differentiable, is

(1) $(-\infty, \infty)\sim\{0\}$ (2) $(-\infty, \infty)\sim\{1\}$

(3) $(-\infty, \infty)\sim\{-1\}$ (4) $(-\infty, \infty)$

41. $f(x)=\begin{cases} \tan^{-1} x, & |x|<1 \\ \frac{x^2-1}{4}, & |x|<1 \end{cases}$, then $f(x)$ is differentiable at

(1) $(-\infty,\infty)\sim\{1\}$ (2) $(-\infty,\infty)\sim\{-1\}$

(3) $(-\infty,\infty)\sim\{1,-1\}$ (4) $(-\infty,\infty)\sim\{-1, 0,1\}$

42. $f(x)=[\sin x]$, where [.] denotes the greatest integer function, is continuous at

(1) $\frac{\pi}{2}$ (2) π

(3) $\frac{3\pi}{2}$ (4) 2π

43. $f(x)=[\tan^{-1} x]$, where [.] denotes the greatest integer function, is discontinuous at

(1) $\frac{\pi}{4}, -\frac{\pi}{4}$ and 0 (2) $\frac{\pi}{3}, -\frac{\pi}{3}$ and 0

(3) $\tan 1, -\tan 1$ and 0 (4) none of these

44. $f(x)=\begin{cases} \sin\{x\}, & x<1 \\ \cos x+a, & x\ge 1 \end{cases}$, where [.] denotes the fractional part. If $f(x)$ is continuous at $x=1$, then

(1) $a=\sin 1$ (2) $a=\cos 1-\sin 1$

(3) $a=\cos 1$ (4) $a=\sin 1-\cos 1$

45. $f(x)=|x^{2n+1}|\ n\in N$, then

(1) $f(x)$ is continuous but non-differentiable at $x = 0$

(2) $f(x)$ is differentiable at $x = 1$

(3) $f(x)$ is discontinuous at $x = 0$

(4) none of the above

46. Value of $f(0)$ so that $f(x)=\frac{1}{x^2}(1-\cos(\sin x))$ can be made continuous at $x = 0$, is equal to

(1) $\frac{1}{2}$ (2) 2

(3) 8 (4) 4

47. $f(x)=\begin{cases} x, & x \text{ is rational} \\ 1-x, & x \text{ is irrational} \end{cases}$ then at $x=\frac{1}{2}$, $f(x)$ is

(1) continuous but non-differentiable

(2) discontinuous

(3) differentiable

(4) none of the above

48. $f(x)=\begin{cases} \frac{x}{2x^2+|x|}, & x\neq 0 \\ 1, & x=0 \end{cases}$ then $f(x)$ is

(1) continuous but non-differentiable at $x = 0$

(2) differentiable at $x = 0$

(3) discontinuous at $x = 0$

(4) none of the above

49. $f(x)=\begin{cases} x+a\sqrt{2}\sin x, & 0\le x<\frac{\pi}{4} \\ 2x\cot x+b, & \frac{\pi}{4}\le x<\frac{\pi}{2} \\ a\cos 2x-b\sin x, & \frac{\pi}{2}\le x\le \pi \end{cases}$ is continuous in $[0, \pi]$, then

(1) $a=-\frac{\pi}{6}, b=-\frac{\pi}{12}$ (2) $a=\frac{\pi}{6}, b=\frac{\pi}{12}$

(3) $a=\frac{\pi}{6}, b=-\frac{\pi}{12}$ (4) $a=-\frac{\pi}{16}, b=\frac{\pi}{12}$

50. $f(x)=[x]+\sqrt{\{x\}}$, where [.] and {.} denote the greatest integer function and fractional part respectively, then

(1) $f(x)$ is continuous but non-differentiable at $x = 1$

(2) $f(x)$ is differentiable at $x = 1$

(3) $f(x)$ is discontinuous at $x = 1$

(4) none of the above

51. If the first derivative of

$f(x)=\log_2\left(\log_3\left(\log_{1/2}(\cos x+a)\right)\right)$ exists

for all real values of x, then

(1) $a\in\left(1,\frac{3}{2}\right)$ (2) $a\in\left(\frac{1}{2},1\right)$

(3) $a\in\left(1,\frac{5}{2}\right)$ (4) none of these

52. If $f(x)=\begin{cases} ax^2+1, & x\le 1 \\ x^2+ax+b, & x>a \end{cases}$ is differentiable at $x = 1$, then

(1) $a=1, b=1$ (2) $a=1, b=0$

(3) $a=2, b=0$ (4) $a=2, b=1$

53. Which of the following statement is incorrect ?

(1) If $f'(x)>0\,\forall\, x\in$ domain, then $f(x)$ must be one-one

(2) If $f'(x)>0\,\forall\, x\in$ domain, then $f(x)$ must be one-one

(3) If $|f(x)|$ be contiuous at x = a, then $f(x)$ is also continuous at x = a

(4) If $f(x)$ is continuous at x - a, $f(a)=2$ and $x=a$ is the point of local of $f(x)$, where [.] denotes greatest integer function, is also continuous at x = a.

54. $f(x)=[\sin x]+[\cos x],\ x\in[0,2\pi]$, where [.] denotes the greatest integer function. Total number of points where $f(x)$ is non-differentiable, is equal to

(1) 2 (2) 3

(3) 5 (4) 4