

TEST -3

CLASS B.A-2/B.SC-2

SEQUENCES AND SERIES

1. By definition show that $\lim_{n \rightarrow \infty} \frac{n^2+1}{2n^2+3} = \frac{1}{2}$.
2. Show that $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \dots + \frac{1}{(2n)^2} \right] = 0$.
3. Show that the seq. $\{x_n\}$ defined by $x_1 = \sqrt{2}, x_{n+1} = \sqrt{2+x_n}$ is bdd, monotonic and cgt to 2.
4. Prove that $\lim_{n \rightarrow \infty} \frac{n}{(n!)^{\frac{1}{n}}} = e$.
5. Show that the seq $\{x_n\}$ where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ does not cgt, by showing that it is not a cauchy seq.
6. Discuss the cgt or div of the series $\sum (\sqrt[3]{n^3+1} - n)$.
7. Discuss the cgt or div of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ $p > 0$
8. Test the cgt or div of the series $\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \dots, x > 0$.
9. Discuss the cgt or div of the series $\sum \frac{a^n}{a^n + x^n}, a > 0, x > 0$.
10. Discuss the cgt or div of the series $1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \dots, x > 0$.