TOPICS COVERED

SOLUTION OF TRIANGLES

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1. If two sides of a triangle are the roots of the equation $4x^2-\left(2\sqrt{6}\right)x+1=0$ and the included angle is 60°, then the third side is
   (1) $\sqrt{3}$ (2) $\sqrt{3}/2$
   (3) $1/\sqrt{3}$ (4) $2\sqrt{3}$

2. In a triangle, if the sum of two sides is x and their product is y such that $(x+z)(x-z)=y$ where z is the third side of the triangle then the triangle is
   (1) equilateral (2) right angled
   (3) obtuse angled (4) none of these

3. In a triangle ABC if $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$, then the which of the following is not true
   (1) $a\tan A=b\tan B=c\tan C$
   (2) $\frac{\tan A}{a}=\frac{\tan B}{b}=\frac{\tan C}{c}$
   (3) $\tan A+\tan+\tan C=3\sqrt{3}$
   (4) $\tan A\tan B\tan C=abc$

4. In a triangle ABC, cos A cos B + sin A sin B sin C = 1, then the triangle is
   (1) equilateral (2) right angled isoscels
   (3) obtuse angled isosceles
   (4) none of these

5. In a triangle ABC, if D is the middle point of BC and AD is perpendicular to AC, then cos B =
   (1) 2 b/a (2) – b/c
   (3) $\left(b^2+c^2\right)/ca$ (4) $\left(c^2+a^2\right)/ca$

6. In the besector of angle A of the triangle ABC makes an angle θ with BC, then $\sin\theta=$
   (1) $\cos\frac{B-C}{2}$ (2) $\cos\frac{B+C}{2}$
   (3) $\cos(B+A/2)$ (4) $\cos(C+A/2)$

7. If the angles of a triangle ABC satisfy the equation $81^{\sin^2 x}+81^{\cos^2 x}=30$, then the triangle can not be
   (1) eqilateral (2) isosceles
   (3) obtuse angled (4) right angled

8. In a triangle ABC if $\sin A\sin B=ab/c^2$, then the triangle is
   (1) equilateral (2) right angled
   (3) obtuse angled (4) none of these

9. If the sides a, b, c of a triangle are in G.P. and largest angle exceeds the smallest by 60°, then cos B =
   (1) 1 (2) $\left(\sqrt{13}-1\right)/4$
   (3) $1/2$ (4) $\left(1-\sqrt{13}\right)/4$

10. If $n, n+1, n+2$; where n is any natural number, represent the sides of a triangle ABC in which the largest angle is twice the smallest, then n =
    (1) 1 (2) 2
    (3) 3 (4) 4

11. If the median of the triangle ABC through A is perpendicular to AB, then *tan A* + *2tan B* =
    (1) *tan C* (2) *sin C*
    (3) *cos C* (4) none of these

12. If in a triangle ABC, $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}=k$ then
    (1) $26-\sec^3 C$ (2) $32-\sec^2 B$
    (3) $44-\sec^3 A$ (4) 0

13. In a triangle the sides are 3, 4, 5 units, then the
    $$\begin{vmatrix} -1/k & 25 & 19 \\ 25 & -1/k & 7 \\ 19 & 7 & -1/k \end{vmatrix}$$
    is equal to circumradius is

(1) 2 (2) 2.5
(3) 3 (4) 3.5

14. In a triangle ABC, the sides are 13, 14, 15 units, then the in-radius *r* is
(1) 3 (2) 4
(3) 5 (4) 7

15. If ABC is not a right angled triangle, then which of the following is possible
(1) $\sin 2A+\sin 2B+\sin 2C = 4\cos A\cos B\sin C$
(2) $\cos 2A+\cos 2B+\cos 2C=-3/2$
(3) $\sin A+\sin B+\sin C = 2\sqrt{2}\cos(A/2)\cos(B/2)$
(4) $\cos A+\cos B+\cos C=1$

16. In a triangle ABC, if $\cos A+2\cos B+\cos C=2$, then a, b, c are in
(1) A.P. (2) G.P.
(3) H.P. (4) none of these

17. In a triangle ABC, if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$, then $\sin A\sin B+\sin B\sin C+\sin C\sin A=$
(1) 0 (2) 9/4
(3) 1
(4) $\cos^2 A+\cos^2 B+\cos^2 C$

18. In a triangle ABC, if $a+b=3c$, then $\cos A+\cos B=$
(1) 3 cos C (2) 3 sin C
(3) 3 cos (A-B) (4) 3 - 3 cos C

19. In a triangle ABC, if A =18°, *b* - *a* = 2, *ab* = 4, then the triangle is
(1) acute angled (2) right angled
(3) obtuse angled (4) isosceles

20. If in a triangle ABC, C = 60°, then $\frac{1}{a+c}+\frac{1}{b+c}$ is equal to
(1) $\frac{a+b+c}{ab}$ (2) $\frac{a-b+c}{ab}$
(3) $\frac{3}{a+b+c}$ (4) $\frac{3}{a+b-c}$

21. If O is point inside the triangle ABC such that $\angle OBC=A/2, \angle OCA=B/2, \angle OAB=C/2$, then
$$\frac{\sin(A-(C/2))\sin(B-(A/2))\sin(C-(B/2))}{\sin(A/2)\sin(B/2)\sin(C/2)},$$
is equal to
(1) $\cos(A/2)\cos(B/2)\cos(C/2)$
(2) $\sin A\sin B\sin C$
(3) 1 (4) $\cos A\cos B\cos C$

22. If in a triangle ABC, $5\cos C+6\cos B=4$ and $6\cos A+4\cos C=5$ then $\tan(A/2)\tan(B/2)$ is equal to
(1) 2/3 (2) 3/2
(3) 1/5 (4) 5

23. If a triangle ABC, $\frac{a(a+c-b)}{b(b+c-a)}$ is equal to
(1) $\frac{1-\cos A}{1-\cos B}$ (2) $\frac{1+\cos A}{1+\cos B}$
(3) $\frac{\cos^2(A/2)}{\sin^2(B/2)}$ (4) $\frac{\sin^2 A}{\sin^2 B}$

24. In a triangle ABC if $\frac{s-a}{11}=\frac{s-b}{12}=\frac{s-c}{13}$, then $\tan^2(A/2)=$
(1) 143/432 (2) 13/33
(3) 11/39 (4) 12/37

25. The perimeter of a triangle right angled at C is 70, and the in-radius is 6, than | *a* – *b* | =
(1) 1 (2) 2
(3) 8 (4) 9

26. If for a triangle ABC, *a*,*b* and A are given , then which of the following gives us two such triangles
(1) $a<b\sin A$ (2) $a=b\sin A$
(3) $a>b\sin A$ *and* $a<b$

27. In a triangle ABC, if B = 30° and $c=\sqrt{3}b$, then A can be equal to
(1) 45° (2) 60°
(3) 90° (4) 120°

28. If in a triangle ABC sines of angles A and B satisfy the equation $4x^2 - 2\sqrt{6} + 1 = 0$, then $\cos(A-B)$ is equal to

(1) 0 (2) 1/2
(3) $1/\sqrt{2}$ (4) $\sqrt{3}/2$

29. If a triangle ABC, A= 2B, then $a^2 - b^2$ is equal to

(1) ab (2) bc
(3) ca (4) none of these

30. In a triangle ABC, if $\frac{a^2+b^2}{a^2-b^2}$ sin (A – B) = 1 and the triangle is not right angled, then $\cos(A-B)$ is equal to

(1) $\sin\left(\frac{C}{2}-\frac{\pi}{4}\right)$ (2) $\cos\left(\frac{C}{2}-\frac{\pi}{4}\right)$
(3) $\tan\left(\frac{C}{2}-\frac{\pi}{4}\right)$ (4) $\cot\left(\frac{C}{2}-\frac{\pi}{4}\right)$

31. In a triangle ABC, if the median AD makes an angle θ with AC and AB = 2AD then $\sin\theta =$

(1) sin A (2) sin B
(3) sin C (4) none of these

32. In a triangle ABC, if $Rr(\sin A + \sin B + \sin C) = 96$, then area of the triangle i units is equal to

(1) 24 (2) 48
(3) 96 (4) 192

33. A quadrilateral ABCD in which AB = a, BC = b CD = c and DA = d is such that one circle can be insribed in it and another circle circumscribed about it, then cos A =

(1) $\frac{ad-bc}{ad+bc}$ (2) $\frac{ab-cd}{ab+cd}$
(3) $\frac{ad+bc}{ad-bc}$ (4) $\frac{ab+cd}{ab-cd}$

34. In a triangle ABC if $\cot\frac{A}{2}\cot\frac{B}{2} = c$, $\cot\frac{B}{2}\cot\frac{C}{2} = a$ and $\cot\frac{C}{2}\cot\frac{A}{2} = b$, then

$\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} =$

(1) – 1 (2) 0
(3) 1 (4) 2

35. In tangents of two angles of a triangle satisfy the equation $x^2 + px - 1$, then the triangle is obtuse angled

(1) if $p > 1$ (2) if $p < 1$
(3) for finite values of p
(4) for all values of p

36. The distance of the incentre of the triangle ABC from A is

(1) $4R\sin(A/2)$
(2) $4R\sin[(B+C)/2]$
(3) $4R\sin(B/2)\sin(C/2)$
(4) none of these

37. If the area of the triangle ABC is $a^2 - (b-c)^2$, then its circumradius R =

(1) $(a/6)\sin^2(A/2)$
(2) $(a/16)\operatorname{cosec}^2(A/2)$
(3) $(b/16)\sin^2(B/2)$
(4) $(c/16)\sin^2(C/2)$

38. In a triangle ABC, $\frac{\cos B + \cos C}{1-\cos A} =$

(1) $\frac{b+c}{1-a}$ (2) $\frac{bc}{1-a}$
(3) $\frac{b+c}{a}$ (4) $\frac{a}{b+c}$

39. In a triangle ABC, $\frac{r_1 + r_2}{1+\cos C} =$

(1) $2ab/c\Delta$ (2) $(a+b)/c\Delta$
(3) $abc/2\Delta$ (4) abc/Δ^2

40. In a triangle ABC, $(a^2 - b^2 - c^2)$ tan A + $(a^2 - b^2 + c^2)$ tan B is equal to

(1) $\left(a^2+b^2-c^2\right)\tan C$

(2) $\left(a^2+b^2+c^2\right)\tan C$

(3) $\left(b^2+c^2-a^2\right)\tan C$

(4) none of these

41. In a triangle ABC, if cot $A=\left(x^3+x^2+x\right)^{1/2}$ $\cot B=\left(x+x^{-1}+1\right)^{1/2}$ and cot C = $\cot B=\left(x^{-3}+x^{-2}+x^{-1}\right)^{-1/2}$ then the triangle is

(1) isosceles (2) obtuseangled
(3) right angled (4) none of these

42. In a triangle ABC, if a is the arithmetic mean and b, c $(b\neq c)$ are the two geometric means between any two positive real numbers then $\dfrac{\sin^3 B+\sin^3 C}{\sin A\sin B\sin C}$ is equal to

(1) 0 (2) 1
(3) 2 (4) 4

43. If D is a point of the base BC of an isoceles triangle ABC such that AD is perpendicular to AC, then a =

(1) 2 b (2) 2b cos B
(3) $\dfrac{1+\cos A}{\sin B}AD$ (4) $\dfrac{1-\sin A}{\cos B}AD$

44. In a triangle ABC $r_1r_2+rr_3=$

(1) $\dfrac{ab}{c}$ (2) abc
(3) ab (4) $(a+b)/c$

45. If d_1,d_2,d_3 are the diameters of the three escribed circles of a triangle ABC, then $d_1\,d_2+d_2\,d_3+d_3\,d_1=$

(1) $ab+bc+ca$ (2) $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}$
(3) $(a+b+c)^2$ (4) none of these

46. If in a triangle ABC, $r_1=2r_2=3r_3;D$ is the mid-point of BC, then $\cos\angle ADC$ is equal to

(1) 7/25 (2) –7/25
(3) 24/25 (4) – 24/25

47. In a triangle ABC, if A,B,C are in A.P. and b : c = $\sqrt{3}:\sqrt{2}$, then

(1) $\sin(2C-A)=\sin(B/4)$

(2) $\sin(A-C)=\sin(B/2)$

(3) $\sin(A+C)=\cos 2B$

(4) $\cos(A-C)=\sin(B/2)$

48. The angles of a triangle ABC satisfy the relations 3B – C = 30° and A+2B = 120°.If the perimeter of the triangle is $2\left(3+\sqrt{3}+\sqrt{2}\right)$, the largest side of the triangle is

(1) $\sqrt{3}+1$ (2) $2+2\sqrt{3}$
(3) $2\sqrt{2}$ (4) 4

49. If A is the area and 2 s,the sum of three sides of a triangle, then

(1) $A\le\dfrac{s^2}{3\sqrt{3}}$ (2) $A\le\dfrac{s^2}{2}$
(3) $A>\dfrac{s^2}{\sqrt{3}}$ (4) none of these

50. In a triangle ABC, if $a=5,b=4$ and c = 3, D and E are the points on BC such that BD = DE = EC. If $\angle DAE=\theta$, then $\tan\theta$ =

(1) 3/8 (2) 2/3
(3) 18/25 (4) $1/\sqrt{3}$

51. In a triangle ABC,the sides a, b, c are respectively 13,14,15.If r_1 is the radius of the escribed circle touching BC and the sides AB and AC produced, then r_1 is equal to

(1) 10.5 (2) 12
(3) 14 (4) 4

52. The distance of the orthocentre of $\triangle ABC$ from its vertex A is

(1) 2R sin A (2) 2R sin B cos B
(3) 2R cos A (4) 2R sin B sin C

53. If R, r denote respectively the circum-radius and in-radius of a triangle, then the distance between the circumcentre and the incentre of the triangle is

(1) 2Rr (2) $\sqrt{R^2 - 2Rr}$

(3) $\sqrt{R^2 + 2Rr}$ (4) R + r

54. If R is the circum radius of the triangle ABC and r_1 is the radius of the escribed circle which touches BC and the two sides AB and AC produced, then the distance of the circumcentre from the centre of the escribed circle is

(1) $2Rr_1$ (2) $\sqrt{R^2 - 2Rr_1}$

(3) $\sqrt{R^2 + 2Rr_1}$ (4) $R + r_1$

55. If R is the circum-radius of a triangle ABC then the distance of the orthocentre of the triangle from its circumcentre is

(1) $\sqrt{R^2 - 8R^2 \sin A \cos B \cos C}$

(2) $\sqrt{R^2 - 8R^2 \cos A \sin B \cos C}$

(3) $\sqrt{R^2 - 8R^2 \sin A \sin B \sin C}$

(4) $\sqrt{R^2 - 8R^2 \cos A \cos B \cos C}$

EXERCISES

1. In a triangle ABC if $\frac{a}{1}=\frac{b}{\sqrt{3}}=\frac{c}{2}$ then
(1) A + B - C =90°
(2) The triangle is acute angled
(3) A,B,C are in A.P.
(4) the triangle is obtuse angled

2. In a triangle ABC, if $\tan\frac{B-C}{2}=\frac{3}{5}\cot\frac{A}{2}$, then D =
(1) $2b^2\sin A$ (2) $2c^2\sin A$
(3) $(1/8)c^2\sin A$ (4) $2c^2\cos A$

3. In an isosceles triangle with base angle α and lateral side 4, Rr =
(1) $8\cos\alpha$ (2) $\frac{8\cos\alpha}{1-\cos\alpha}$
(3) $\frac{8\cos\alpha}{1+\cos\alpha}$ (4) $8\cos\alpha(1-\cos\alpha)$

4. In a triangle ABC, if *s* - *a*, *s* - *b*, *s* - *c* are in G.P., then $\frac{\sin^2 A+\sin^2 C}{\sin A+\sin C}=$
(1) sin B (2) cos B
(3) sin [(A + C)/2] (4) sin[(A - C)/2]

5. a right angled triangle of maximum area is inscribed in a circle of radius R, the radius of the incircle is
(1) $\sqrt{2}R$ (2) $(\sqrt{2}-1)R$
(3) R/2 (4) none of these

6. If *x*, *y*, *z* are perpendiculars from the angular points of a $\triangle ABC$ upon the opposite sides *a*, *b*, *c* respectively, then $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}$ is equal to
(1) $b\cos B+c\cos C+a\cos A$
(2) $2R(\sin^2 A+\sin^2 B+\sin^2 C)$
(3) $2R(\cos^2 A+\cos^2 B+\cos^2 C)$
(4) none of these

7. If a chord of length 1 unit subtends an angle θ at the circumference of a circle whose radius is R then R sin θ =
(1) 2 (2) 1/2
(3) 1/3 (4) 1/4

8. In a triangle ABC, AD is perpendicular to BC and DE is perpendicular to AB, then area of ΔADE =
(1) $(c^2/4)\cos^2 B\sin 2B$
(2) $(c2/4)\sin^2 B\sin 2B$
(3) $(c^2/4)\sin 2B$ (4) $(b^2/4)\sin 2C$

9. In a triangle ABC $\frac{bc\sin^2 A}{\cos A+\cos B\cos C}=$
(1) b^2+c^2 (2) bc
(3) a^2 (4) a^2+bc

10. If the sides of a triangle are 17, 10, 21, then the radius of the circum-circle is approximately
(1) 10.5 (2) 10.6
(3) 10.7 (4) 10.8

11. In a triangle PQR, sin P, sin Q, sin R are in A.P., then
(1) the altitudes are in A.P.
(2) the altitudes are in H.P.
(3) the medians are in A.P.
(4) the medians are in H.P.

12. In a triangle ABC, sin A, sin B, sin C are irrational numbers, then
(1) sides of the triangle must be irrational numbers
(2) medians of the triangle must be irrational numbers
(3) altitudes of the triangle must be irratinal numbers
(4) none of these

13. In a triangle ABC, Let $\angle C=\pi/2$, if r is the inradius and R is the circum radius of the triangle, then $2(r+R)$ is equal to
(1) $a+b$ (2) $b+c$
(3) $c+a$ (4) $a+b+c$

14. In a triangle ABC, 2ac sin [(A - B + C)/2] is equal to
(1) $a^2+b^2-c^2$ (2) $c^2+a^2-b^2$
(3) $b^2-c^2-a^2$ (4) $c^2-a^2-b^2$

15. In a triangle ABC, if $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ and $a=2\sqrt{3}$ cm, then the area of the triangle is
(1) $2\sqrt{3}\,cm^2$ (2) $3cm^2$
(3) $4cm^2$ (4) $3\sqrt{3}\,cm^2$

16. If $\cos A=\frac{\sin B}{2\sin C}$, then ΔABC is
(1) equilateral (2) isosceles
(3) right angled (4) none of these

17. In an equilateral triangle the in-radius, circumradius and one of the ex-radii aer in the ratio
(1) 2 : 3: 5 (2) 1 : 2 : 3
(3) 1 : 3 : 5 (4) 3 : 5 : 7

18. If the angles of a Δ are 30° and 45° and the included side is $\left(\sqrt{3}+1\right)$ cm, the area of the triangle is
(1) $(1/2)\left(\sqrt{3}+1\right)cm^2$
(2) $(1/4)\left(\sqrt{3}+1\right)cm^2$
(3) $(1/6)\left(\sqrt{3}+1\right)cm^2$
(4) $(1/8)\left(\sqrt{3}+1\right)cm^2$

19. If α,β,γ are the lengths of the altitudes of a triangle ABC, then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$ is equal to
(1) π/Δ (2) Δ/π
(3) π (4) none of these
where $\pi=\cot A+\cos B+\cot C$ and Δ is the of the triangle

20. In any triangle $ABC,\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}$ is equal to
(1) $\frac{1}{2R}-\frac{1}{r}$ (2) $2R-r$
(3) $r-2R$ (4) $\frac{1}{r}-\frac{1}{2R}$

21. In a griangle ABC $\frac{b^2-c^2}{a}\cos A+\frac{c^2-a^2}{b}\cos B+\frac{a^2-b^2}{c}\cos C$ is equal to
(1) 0 (2) 1
(3) $a^2+b^2+c^2$ (4) a b c

22. If the angles, A, B, C of triangle ABC are in arithmetical progression then
(1) $\tan A+\tan C-\sqrt{3}\tan A\tan C=\sqrt{3}$
(2) $\tan A+\tan C+\sqrt{3}\tan A\tan C=\sqrt{3}$
(3) $\tan A+\tan C-\sqrt{3}\tan A\tan C=-\sqrt{3}$
(4) $\tan A+\tan C+\sqrt{3}\tan A\tan C=-\sqrt{3}$

23. In a griangle $ABC\,if\,\frac{\cos A}{a}=\frac{\tan C}{c}$, then sin (B+C) is equal to
(1) cos B cos C (2) coa A cos C
(3) cos A cos B (4) sin B sin C

24. If the angles of a triangle are in the ratio 1: 3 : 5 and θ denotes the smallest angle, then the ratio of the largest side to the smallest side of the triangle is
(1) $\frac{\sqrt{3}\sin\theta+\cos\theta}{2\sin\theta}$ (2) $\frac{\sqrt{3}\cos\theta-\sin\theta}{2\sin\theta}$
(3) $\frac{\cos\theta+\sqrt{3}\sin\theta}{2\sin\theta}$ (4) $\frac{\sqrt{3}\cos\theta+\sin\theta}{2\sin\theta}$

25. In a triangle ABC, $(b+c)/a$ is equal to
(1) $\frac{\sin[(B-C)/2]}{\sin[(B+C)/2]}$ (2) $\frac{\cos[(B-C)/2]}{\cos[(B+C)/2]}$
(3) $\frac{\sin[(B+C)/2]}{\sin[(B-C)/2]}$ (4) $\frac{\cos[(B+C)/2]}{\cos[(B-C)/2]}$

26. In a triangle ABC, 1– tan (A/2) tan (B/2) is equal to

(1) $\frac{2a}{b+c-a}$ (2) $\frac{2b}{c+a-b}$

(3) $\frac{2c}{a+b-c}$ (4) $\frac{2c}{a+b+c}$

27. If in a triangle ABC, b + c = 3*a*, then cot (B/2) cot (C/2) is equal to

(1) 1/2 (2) 1

(3) 2 (4) non of these

28. The angles of a triangle ABC are in A.P. The largest is twice the smallest and the median to the largest side divides the angle at the vertex in the ratio 2 : 3. If length of the median is $2\sqrt{3}\,cm$, length of the largest side is

(1) 2 sin 32° (2) 2 sin 48°

(3) 8 sin 32° (4) $\sqrt{3}$ sin 40°

29. The expression $\frac{(a+b+c)(b+c-a)}{(c+a-b)(a+b-c)}$ is equal to

(1) $\cos^2(A/2)$ (2) $\sin^2(A/2)$

(3) $\cot^2(A/2)$ (4) $\tan^2(A/2)$

30. If the median AD of a triangle ABC divides the angle $\angle BAC$ in the ratio 1 : 2, then sinB/sinC is equal to

(1) $2\cos(A/3)$ (2) $(1/2)\sec(A/3)$

(3) $(1/2)\sin(A/3)$ (4) $2\,\mathrm{cosec}\,(A/3)$

31. In a triangle ABC, $b\sin^2(A/2)+a\sin^2(B/2)$ is equal to

(1) s – c (2) s + c

(3) $s-(a+b)$ (4) none of these

32. In a triangle ABC , if $c=(a-b)\sec\theta$, then $\tan\theta$ is equal to

(1) $\frac{2\sqrt{ab}}{a-b}\sin\frac{C}{2}$ (2) $\frac{2\sqrt{ab}}{a-b}\tan\frac{C}{2}$

(3) $\frac{2\sqrt{ab}}{a-b}\cos\frac{C}{2}$ (4) $\frac{2\sqrt{ab}}{a-b}\sec\frac{C}{2}$

33. The vertical angle of a triangle is divided into two parts, such that the tangent of one part is 3 times the tangent of the other and the difference of these parts is 30°, then the triangle is

(1) isosceles (2) right angled

(3) obtuse angled (4) none of these

34. In a triangle ABC, $(b-c)\cos(A/2)$ is equal to

(1) $a\cos[(B-C)/2]$ (2) $a\sin[(B-C)/2]$

(3) $a\cos[(B+C)/2]$ (4) $a\sin[(B+C)/2]$

35. In a cyclic quadrilateral ABCD; a, b, c, d denote the length of the sides AB, BC, CD and DA respectively, then cos A is equal to

(1) $\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$ (2) $\frac{b^2+c^2-d^2-a^2}{2(bc+da)}$

(3) $\frac{c^2+d^2-a^2-b^2}{2(cd+ab)}$ (4) $\frac{d^2+a^2-b^2-c^2}{2(da+bc)}$

36. In a triangle ABC, $\sin A+\sin B+\sin C$ is maximum when the triangle is

(1) right angled (2) isosceles

(3) equilateral (4) obtuse angle

37. In a triangle ABC, if $\tan(A/2)=p, \tan(B/2)=q$, then $\frac{2(p+q)(1-pq)}{(1+p^2)(1+q^2)}$ is equal to

(1) sin A (2) sin B

(3) sin C (4) sin A + sin B

38. In a triangle ABC, $a^2\cos 2B+b^2\cos 2A$ is equal to

(1) $a^2+b^2+4ab\sin A\sin B$

(2) $a^2+b^2-4ab\sin A\sin B$

(3) $a^2+b^2+4ab\cos A\sin B$

(4) $a^2+b^2-4ab\cos A\sin B$

39. If the angles A and B of the triangle ABC satisfy the relation $\sin A+\sin B=\sqrt{3}(\cos B-\cos A)$ then they

differ by

(1) $\pi/6$ (2) $\pi/3$

(3) $\pi/4$ (4) $\pi/2$

40. In a triangle ABC, right angled at C, the sum of the tangents of the other two angles is 169/60. The tangent of the larger of these two is equal to

(1) 5/12 (2) 12/5

(3) 12/13 (4) 13/5

41. The sides of a triangle are 17, 25, 28, The length of the largest altitude is

(1) 15 (2) 84/5

(3) 420/17 (4) 210/17

42. Two angles of a triangular field are 22.5° and 45°, and the length of the sides opposite to the later is 200 metres. The area of the field is

(1) $50^2\,m^2$ (2) $100^2\,m^2$

(3) $200^2\,m^2$ (4) $100\,m^2$

43. In a triangle ABC if $A=\pi/4$ and tan B tan C = k, then k must satisfy

(1) $k^2-6k+1\geq 0$ (2) $k^2-6k+1=0$

(3) $k^2-6k+1\leq 0$

(4) $3-2\sqrt{2}<k<3+2\sqrt{2}$

44. The sides of a triangle are in the ratio $5:8:11$. If θ denotes the largest angle of the triangle then $\tan^2(\theta/2)$ is equal to

(1) 1/21 (2) 7/48

(3) 7/3 (4) 7/12

45. In a triangle ABC if

$\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a}{bc}$, then A is

(1) an acute angle (2) an obtuse angle

(3) a right angle (4) equal to B - C

46. If one base angle of a triangle is five times the other and half the angle at the vertex, then base of the triangle is equal to

(1) the height (2) half the height

(3) twice the height (4) five times the height

47. In a triangle ABC, the median AD is perpendicular to AC; if $b=5, c=11$, then a is equal to

(1) 10 (2) 12

(3) 14 (4) $\sqrt{221}$

48. In a triangle ABC, if $b=7, c=4$ and A = 120° then the length of the bisector of angle A is

(1) 28/11 (2) $28\sqrt{3}/11$

(3) $2\sqrt{3}/11$ (4) $7\sqrt{3}/22$

49. If A, B, C, D are the angles of a quadrilateral, then $\frac{\tan A+\tan B+\tan C+\tan D}{\cot A+\cot B+\cot C+\cot D}$ is equal to

(1) $\tan A\tan B\tan C\tan D$

(2) $\cot A\cot B\cot C\cot D$

(3) $\tan^2 A+\tan^2 B+\tan^2 C+\tan^2 D$

(4) $\sum\tan A\tan B\tan C$

50. If in a triangle ABC, the line joining the circumcentre O and the incentre I is parallel to BC, then

(1) $r=R\cos A$ (2) $r=R\sin A$

(3) $R=r\cos A$ (4) $R=r\sin A$

51. If the sides of a triangle are 17, 10 and 21, then the largest ex-radii of the triangle is

(1) 6 (2) 12

(3) 28 (4) none of these

52. If the area of a triangle is 96, and the radii of the escribed circles are 8, 12, 24, then the largest side of the triangle is

(1) 16 (2) 18

(3) 20 (4) 32

53. If R is the circumradius of a triangle ABC then the area of its pedal triangle is

(1) $(1/2)R^2\sin A\sin B\sin C$

(2) $(1/2)R^2\sin 2A\sin 2B\sin 2C$

(3) $(1/2)R^2\cos A\cos B\cos C$

(4) $(1/2)R^2\cos 2A\cos 2B\cos 2C$

54. If R is the circumradius of the triangle ABC,then the circumradius of its pedal triangle is

(1) $\sqrt{R}$ (2) R/2

(3) R/3 (4) none of these

55. If S is the circumcentre of the triangle whose circumradius is $R; I, I_1, I_2, I_3$ are respectively the incentre and excentres of the triangle, then $SI^2 + SI_1^2 + SI_2^2 + SI_3^2$ is equal to

(1) $4R^2$ (2) $8R^2$

(3) $12R^2$ (4) none of these

56. If I is the incentre of the triangle ABC, then $aAI^2 + bBI^2 + cCI^2$

(1) abc (2) $a^2 + b^2 + c^2$

(3) $(1/2)(a+b+c)$ (4) $a^2b^2c^2$

57. Let ABC be a triangle I_1, I_2, I_3 its ex-centres : then orthocentre of the *Ex-central triangle* $I_1 I_2 I_3$ is the

(1) centroid (2) circumcentre

(3) in-centre (4) orhtocentre

58. If R is the circumradius of the triangle ABC then the circumradius of the ex-central triangle is equal to

(1) R/2 (2) 2R

(3) R^2 (4) $\sqrt{R}$

59. If I is the incentre of a triangle whose in radius and circumradius are r and R respectively; $I_1 I_2 I_3$ is its ex-central triangle, then $II_1 . II_2 . II_3$ is equal to

(1) R^2r (2) 16 R^2

(3) Rr^2 (4) $16Rr^2$

60. If the radius of the circumcircle of a triangle is twice the radius of the in-circle of the triangle then the triangle is

(1) right-angled (2) isosceles

(3) equilateral (4) obtuse angled

TOPICS COVERED

HEIGHTS AND DISTANCES

1. A flagstaff stand in the centre of a rectangular field whose diagonal is 1200 m, and subtends angles 15° and 45° at the mid points of the sides of the field. The height of the flagstaff is

(1) 200 m (2) $300\sqrt{2+\sqrt{3}}\ m$

(3) $300\sqrt{2-\sqrt{3}}\ m$ (4) 400 m

2. Two flagstaffs stand on a horizontal plane. A and B are two points on the line joining their feet between them. The angles of elevation of the tops the flagstaffs as seen from A are 30° and 60° and as seen from B are 60° and 45°. If AB is 30 m, the distance between the flagstaffs in metres is

(1) $30+15\sqrt{3}$ (2) $45+15\sqrt{3}$

(3) $60-15\sqrt{3}$ (4) $60+15\sqrt{3}$

3. In a cubica hall A B C D P Q R S with each side 10 m G is the centre of the wall B C R Q and T is the mid point of the side AB. The angle of elevation of G at the point T is

(1) $\sin^{-1}(1/\sqrt{3})$ (2) $\cos^{-1}(1/\sqrt{3})$

(3) $\tan^{-1}(1/\sqrt{3})$ (4) $\cot^{-1}(1/\sqrt{3})$

4. Two vertical poles 20 m and 80 m high stand apart on a horizontal plane. The height of the point of intersection of the ines joining the top of each pole to the foot of the other is

(1) 15 m (2) 16 m

(3) 18 m (4) 50 m

5. A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is

(1) $100\sqrt{3}$ (2) $200\sqrt{3}/3$

(3) $100\sqrt{3}/3$ (4) $200\sqrt{3}$

6. A pole stands vertically, inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the

(1) centroid (2) circumcentre

(3) incentre (4) orthocentre

7. A man observes that the angle of elevation of the top of a tower from a point P on the ground is θ. He moves a certain distance towards the food of the tower and finds that the angle elevation of the top has doubled. He further moves a distance 3/4 of the previous and finds that the angle of elevation is three times that at P. The angle θ is given by

(1) $\sin\theta=\sqrt{5/12}$ (2) $\cos\theta=\sqrt{5/12}$

(3) $\sin\theta=3/4$ (4) $\cos\theta=3/8$

8. A and B are two points 30 m apart in a line on the horizontal plane through the foot of a tower lying on opposite sides of the tower. If the distance of the of the tower from A and B are 20 m and 15 m respectively, the angle of elevation of the top of the tower at a is

(1) $\cos^{-1}(43/48)$ (2) $\sin^{-1}(43/48)$

(3) $\cos^{-1}(29/36)$ (4) $\sin^{-1}(29/36)$

9. A vertical pole subtends P on the ground. The angle subtended by the upper half of the pole at the P is

(1) $\tan^{-1}(1/4)$ (2) $\tan^{-1}(2/9)$

(3) $\tan^{-1}(1/8)$ (4) $\tan^{-1}(2/3)$

10. An aeroplane flying at a height of 3000 m above the ground passes vertically above another at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane the ground is

(1) $1000\sqrt{3}\,m$ (2) $1000/\sqrt{3}\,m$

(3) 500 m (4) $1500\left(\sqrt{3}+1\right)m$

11. A pole of height h stands at one corner of a park in the shape of an equilateral triangle. If α is the the angle which the pole subtends at the midpoint of the opposite side, the length of each side of the park is

(1) $\left(\sqrt{3}/2\right)h\cot\alpha$ (2) $\left(2/\sqrt{3}\right)h\cot\alpha$

(3) $\left(\sqrt{3}/2\right)h\tan\alpha$ (4) $\left(2/\sqrt{3}\right)h\tan\alpha$

12. If each side of length a of an equilateral triangle subtends an angle of 60° at the top of tower h situated at the centre of the triangle, then

(1) $3a^2=2h^2$ (2) $2a^2=3h^2$

(3) $a^2=3h^2$ (4) $3a^2=h^2$

13. A pole 50 m high stands a building 250 m high. To an observer at a height of 300 m, the building and the pole subtend equal angles. The horizontal distance of the observer from the pole is

(1) 25 m (2) 50 m

(3) $25\sqrt{6}\,m$ (4) $25\sqrt{3}\,m$

14. The angle of elevation of the top C of a vertical tower CD of height h from a point a in the horizontal plane is 45° and from a point B at a distance a from A on the line making an angle 30° with AD, it is 60°, then

(1) $a=h\left(\sqrt{3}+1\right)$ (2) $h=a\left(\sqrt{3}+1\right)$

(3) $a=h\left(\sqrt{3}-1\right)$ (4) $h=a\left(\sqrt{3}-1\right)$

15. The angle of elevation of a vertical tower standing inside a triangular field at the vertices of the are each equal to θ. If the length of the sides of the field are 30 m, 50 and 70 m, the height of the tower is

(1) $70\sqrt{3}\tan\theta\,m$ (2) $\left(70/\sqrt{3}\right)\tan\theta\,m$

(3) $\left(50/\sqrt{3}\right)\tan\theta\,m$ (4) $75\sqrt{3}\theta\,m$

16. Two vertical poles of height a and b subtend the same angle 45° at a point on the line joining their feet, the square of the distance between their tops is

(1) $\frac{1}{2}\left(a^2+b^2\right)$ (2) a^2+b^2

(3) $2\left(a^2+b^2\right)$ (4) $(a+b)^2$

17. A monument ABCD stands an a level grond. At point P on the ground the portions AB, AC, AD subtend angles α,β,γ respectively. If AB = a, AC = b, AD=c, AP = x and $\alpha+\beta+\gamma=180^0$ then x^2 is equal to

(1) $\frac{a}{a+b+c}$ (2) $\frac{b}{a+b+c}$

(3) $\frac{c}{a+b+c}$ (4) $\frac{abc}{a+b+c}$

18. In a triangular plot ABC with BC = 7cm, CA=8m and AB = 9m. A lamp post is situated at the middle point E of the side AC and subtends an angle $\tan^{-1}$ 3 at the point B, the height of the lamp post is

(1) 21 m (2) 24 m

(3) 27 m

(4) can not be determined

19. A vertical tower CP subtends the same angle θ, at point B on the horizontal plane through C, the of the tower, and at point A in the vertical plane. If the triangle ABC is equilateral with length of each side equal to 4m, the height of the tower is

(1) $8\sqrt{3}\,m$ (2) $4\sqrt{3}/3\,m$

(3) $4\sqrt{3}\,m$ (4) $8/\sqrt{3}\,m$

20. Two objects at the points P and Q subtend an angle of 30° at a point A. Lengths AR = 20 m and AS = 10 m are measured from A at right angles to AP and AQ respectively. If PQ subtends equal angles of 30°, at R and S, then length of PQ is

(1) $\sqrt{300-200\sqrt{3}}$ (2) $\sqrt{500-200\sqrt{3}}$

(3) $\sqrt{500\sqrt{3}-200}$ (4) $\sqrt{300}$

21. From a ship at sea it is observed that the angle

subtended by feet A and B of two light houses, at the ship is 30°. the ship sails 4 km towards A and this angle is then 48°, the distance of B from the ship at the second observation is

(1) 6.460 km (2) 6.472 km
(3) 6.476 km (4) 6.478 km

22. A man on the ground observes that the angle of elevation of the top of a tower is 68° 11', and a flagstaff 24 m high on the summit of the tower subtends an angle of 2° 10' at the observer's eye. If tan 70° 21' = 2.8 and cot 68° 11' = 0.4 the height of the tower is

(1) 120 m (2) 168 m
(3) 200 m (4) 300 m

23. A statue, standing on the top of a pillar 25 m high subtends an angle whose tangent is 0.125 at a point 60 m from the foot of the pillar. The best approximation for the height of the statue is

(1) 9.28 m (2) 9.29 m
(3) 9.30 m (4) 10 m

24. A tower BCD surmounted by a spire DE stands on a horizontal plane. At the extremity a of horizontal line Ba it is found that BC and DE subtend equal angles. If BC = 3 m, CD = 28 m and DE = 5 m, then BA is equal to

(1) $\sqrt{18\times 93}$ (2) $\sqrt{36\times 93}$
(3) $\sqrt{34\times 93}$ (4) $\sqrt{34\times 36}$

25. From a point on the horizontal plane, the elevation of the top of a hill is 45°. After walking 500 m towards its summit up a slope inclined at an angle of 15° to the horizontal the elevatio is 75°, the height of the hill is

(1) $500\sqrt{6}\,m$ (2) $500\sqrt{3}\,m$
(3) $250\sqrt{6}\,m$ (4) $250\sqrt{3}\,m$

26. A lamp post standing at a point a on a circular path of radius r subtends an angle α at some point B on the path, and AB subtends an angle of 45° at any other point on the path, then height of the loampost is

(1) $\sqrt{2}\,r\cot\alpha$ (2) $\left(r/\sqrt{2}\right)\tan\alpha$
(3) $\sqrt{2}r\tan\alpha$ (4) $\left(1/\sqrt{2}\right)\cot\alpha$

27. A tree is broken by wind, its upper part touches the ground at a point 10 m from the foot of the tree and makes an angle 45° with the ground. The entire length of the tree was

(1) 15 m (2) 20 m
(3) $10\left(\sqrt{2}+1\right)m$ (4) $5\left(\sqrt{3}+2\right)m$

28. From the top of a cliff x m high, the angle of depression of the foot of a tower is found to be double the angle of elevation of the tower. If the height of the tower is h, the angle of elevation is

(1) $\sin^{-1}\sqrt{x/(2-h)}$ (2) $\tan^{-1}\sqrt{3-2h/x}$
(3) $\sin^{-1}\sqrt{2h/x}$ (4) $\cos^{-1}\sqrt{2h/x}$

29. The elevation of a stteple at a place due south of it is 45° and at a place B due west of A the elevation is 15°. If AB = 2a, the height of the steeple is

(1) $a\dfrac{\left(\sqrt{3}-1\right)}{\sqrt{2}}$ (2) $a\dfrac{\left(\sqrt{3}+1\right)}{\sqrt{2}}$
(3) $a\left[3^{\frac{1}{4}}-3^{-\frac{1}{4}}\right]$ (4) $a\left[3^{\frac{1}{4}}+3^{-\frac{1}{4}}\right]$

30. A river flows due NOrth, and a tower stands on its left bank. From a point A upstream and on the same bank as the tower, the elevation of the tower is 60°, and from a point B just opposite A on the other bank the elevation is 45°. If the tower is 360 m high, the breadth of the river is

(1) $120\sqrt{6}\,m$ (2) $240\sqrt{3}\,m$
(3) $240\sqrt{3}\,m$ (4) $240\sqrt{6}\,m$

31. The top of a pole, placed against a wall at an angle α with the horizon, just touches the coping, and when its foot is moved a m, away from the the wall and its angle of inclination is β, it rests on the still of a window;

(1) $a\sin\left((\alpha+\beta)/2\right)$ (2) $a\cos\left((\alpha+\beta)/2\right)$
(3) $a\cot\left((\alpha+\beta)/2\right)$ (4) $a\tan\left((\alpha+\beta)/2\right)$

32. OAB is a triangle in the horizontal plane through the foot P of the tower at the middle point of the side OB of the triangle. If OA = 2m, OB = 6m, AB = 5 m and $\angle AOB$ is equal to the angle subtended by the tower at A then the height of the tower is

(1) $\sqrt{\frac{11\times39}{25\times3}}$ (2) $\sqrt{\frac{11\times39}{25\times2}}$

(3) $\sqrt{\frac{11\times25}{39\times2}}$ (4) none of these

33. If two vertical towrs PQ and RS of lengths *a* and *b* (*a* > *b*) respectively subtend the same angle α at a point a on the line joining their feet P and R in the horizontal plane and angles β *and* γ at another point B on this line nearer the towers on the side of the towers as A, then $\frac{\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$ is equal to

(1) $\frac{b\sin\alpha}{(a-b)\sin\gamma}$ (2) $\frac{(b-1)\sin\gamma}{b\ \sin\alpha}$

(3) $\frac{\sin\gamma}{\sin\alpha}$ (4) $\frac{(b-a)\sin\alpha}{b\sin\gamma}$

34. The angle of elevation of a clound at a height *h* above the level of water in a lake is α, and the angle of depression of its image in the lake is β. The height of the clound above the surface of the lake is

(1) $\frac{h(\cot\alpha+\cot\beta)}{\cot\beta-\cot\alpha}$ (2) $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha-\tan\beta}$

(3) $\frac{h\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ (4) $\frac{h\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$

35. A person standing on the ground observes the angle of elevation of the top of a tower to be 30°. On walking a distance *a* in a certain direction, he finds the elevation of the top to be the same as before. He then walks a distance 5*a*/3 at right angles to his former direction, and finds that the elevation of the top has doubled. The height of the tower is

(1) *a* (2) $\sqrt{85/48}\ a$

(3) $\sqrt{6/5}\ a$ (4) $\sqrt{48/85}\ a$

36. a balloon of radius *r* subtends an angle α at the eye of an observer and the elevation of the centre of the balloon from the eye is β, the height *h* of the centre of the balloon of the balloon is given by

(1) $\frac{r\sin\beta}{\sin\alpha}$ (2) $r\sin b\sin a$

(3) $\frac{r\sin\alpha}{\sin(\alpha/2)}$ (4) $\frac{r\sin\alpha}{\sin(\beta/2)}$

37. A tower PQ subtends an angle α at a point A on the same level as the foot Q of the tower. It also subtends the same angle α at a point B where AB subtends the angle α with AP then

(1) AB = BQ (2) BQ = 2AQ

(3) $\frac{AB}{BQ}=\frac{1}{2}\sin\alpha$ (4) $\frac{AB}{BQ}=\frac{1}{2}\text{cosec}\,\alpha$

38. Two poles of height a and b stand at the centres of two circular plots which touch each other externally at a point and the two poles subtend angles of 30° and 60° repectively at this point, then distance between the centres of these plots is

(1) *a* + *b* (2) $(3a+b)/\sqrt{3}$

(3) $(a+3b)/\sqrt{3}$ (4) $a\sqrt{3}+b$

39. The angle of elevation of the top of a tree at a point B due south of it is 60° at a point C due north of its is 30°. D is a point due north of C where the angle of elevation is 15°, then given $\sqrt{31}\frac{8}{11}$ and BC × CD = $2^2 \times 3^2 \times 19 \times 11$, the height of the tree is

(1) 33 (2) 38

(3) 57 (4) 88

40. *n* poles standing at equal distances on a straight road subtend the same angle α at a point O on the road. If the height of the largest pole is *h* and the distance of the foot of the smallest pole from O is *a*, the distance between two consecutive poles is

(1) $\frac{h\sin\alpha-a\cos\alpha}{(n-1)\sin\alpha}$ (2) $\frac{h\cos\alpha-a\cos\alpha}{(n-1)\cos\alpha}$

(3) $\frac{h\cos\alpha-a\sin\alpha}{(n-1)\sin\alpha}$ (4) $\frac{h\sin\alpha-a\cos\alpha}{(n-1)\cos\alpha}$

EXERCISES

1. The angle of elevataion of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45°. If the angle of elevation of the top of the pillar after completion at the same point is 60°, then the height to be increased for the completion of the pillar in metres is
   (1) $50\sqrt{3}$ (2) $100\sqrt{2}$
   (3) $100\sqrt{3}$ (4) $100\left(\sqrt{3}-1\right)$

2. The angle of elevation of the top of a tower at the top and the foot of a pole 10 m high are 30° and 60° respectively. The height of the tower is
   (1) 15 m (2) 20 m
   (3) $10\sqrt{3}\,m$ (4) $25\sqrt{3}\,m$

3. A tower subtends an angle α at a point A on the ground, and the angle of depression of its foot from a point B just above a and at distance b from a, is β. The height of the tower is
   (1) $b\tan\alpha\tan\beta$ (2) $b\tan\alpha\cot\beta$
   (3) $b\cot\alpha\cot\beta$ (4) $b\cot\alpha\tan\beta$

4. A person walking along a straight road observes that at two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is
   (1) $250\left(\sqrt{3}+1\right)m$ (2) $250\left(\sqrt{3}-1\right)m$
   (3) $500\left(\sqrt{2}+1\right)m$ (4) $500\left(\sqrt{2}-1\right)m$

5. If a flagstaff subtends the same angle at the points A, B, C and d on the horizontal plane through its foot, then ABCD is
   (1) square (2) cyclic quadrilateral
   (3) rectangle (4) none of these

6. From a point on the ground 100 m away from the base of building, the angle of elevation of the top of the building is 60°. Which of the following is the best approximation for the height of the building ?
   (1) 172 m (2) 173 m
   (3) 174 m (4) 175 m

7. From the top of a tower 100 m height, the angles of depression of two objects 200 m apart on the horizontal plane and in a line passing through the foot of the tower and on the same side of the tower are 45° – A and 45° + A, then angle A is equal to
   (1) 15° (2) 22.5°
   (3) 30° (4) 35°

8. An observer finds that the angular elevation of a tower is A, on advancing 3m towards the tower the elevation is 45° and on advancing 2m nearer, the elevation is 90° – A, the height of the tower is
   (1) 1 m (2) 5 m
   (3) 6 m (4) 8 m

9. ABC is a triangular park with all sides equal. If a pillar at A subtends on angle of 45° at C, the angle of elevation of the pillar at D, the middle point of BC is
   (1) $\tan^{-1}\left(\sqrt{3}/2\right)$ (2) $\tan^{-1}\left(2/\sqrt{3}\right)$
   (3) $\cot^{-1}\sqrt{3}$ (4) $\tan^{-1}\sqrt{3}$

10. A kite is flying with the string inclined at 75° to the horizon. If the length of the string is 25 m, the height of the kite is
   (1) $(25/2)\left(\sqrt{3}-1\right)^2$
   (2) $(25/4)\left(\sqrt{3}+1\right)\sqrt{2}$
   (3) $(25/2)\left(\sqrt{3}+1\right)^2$
   (4) $(25/2)\left(\sqrt{6}+\sqrt{2}\right)$

11. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If AP = n AB, then $\tan\beta =$
   (1) $\dfrac{n}{2n^2+1}$ (2) $\dfrac{n}{n^2-1}$
   (3) $\dfrac{n}{n^2+1}$ (4) none of these

12. a man in a boat rowing away from a cliff 150 metres high observes that it takes 2 minutes to change the angle of elevation of the top of the cliff from 60° to 45°. The speed of the boat is

(1) $(1/2)\left(9-3\sqrt{3}\right)km/h$

(2) $(1/2)\left(9+3\sqrt{3}\right)km/h$

(3) $(1/2)\left(9\sqrt{3}\right)km/h$

(4) none of these

13. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°, when he retires 40 metres from the banks he finds the angle at 30°. The breadth of the river is

(1) 40 m (2) 60 m

(3) 20 m (4) 30 m

14. The elevatio of the top of a mountain at each of the three angular points A, B and C of a plane horizontal triangle is α, if BC = a the height of the mountain is

(1) $(a/2)\operatorname{cosec} A\tan\alpha$

(2) $(a/2)\sec A\tan\alpha$

(3) $(a/2)\operatorname{cosec}\alpha\cot A$

(4) $(a/2)\sec\alpha\tan A$

15. From the top of a building of height h, a tower standing on the ground is observed to make an angle θ. If the horizontal distance between the building and the tower is h, the height of the tower is

(1) $\frac{2h\cos\theta}{\sin\theta+\cos\theta}$ (2) $\frac{2h}{1+\cot\theta}$

(3) $\frac{2h}{1+\tan\theta}$ (4) $\frac{2h}{\sin\theta+\cos\theta}$

16. The angles of elevation of the top of a tower standing on a horizontal plane, from two points on a line passing through its foot at distance a and b, respectively, are complementary angles. If the line joining the two points subtends an angle θ at the top of the tower, then if $a > b$

$\sin\theta =$

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$

(3) $\frac{2\sqrt{ab}}{a+b}$ (4) $\frac{2\sqrt{ab}}{a-b}$

17. The upper three-quarters of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and distant 40 m from it. The height of the pole is

(1) 80 m (2) 100 m

(3) 160 m (4) 200 m

18. PQ is avertical tower and A, B, C are three points on a horizontal line through Q, the foot of the tower and on the same side of the tower. If the angles of elevation of the top of the tower from A, B and C are α, β, γ respectively, then AB / BC =

(1) (2)

(3) (4)

19. ABCD is a rectangular park with AB = a. A tower standing at C makes angle α *and* β at A and B respectively, the height of the tower is

(1) $\frac{a}{\sqrt{\cot^2\alpha+\cot^2\beta}}$ (2) $\frac{a}{\sqrt{\cot^2\alpha-\cot^2\beta}}$

(3) $\frac{a\tan\alpha\tan\beta}{\sqrt{\tan^2\beta-\tan^2\alpha}}$ (4) $\frac{a\cot\alpha\cot\beta}{\sqrt{\cot^2\alpha-\cot^2\beta}}$

20. Two circular path of radii a and b intersect at a point O and aB is a common chord of these circles at A and B respectively. Chords OA and OB subtend equal angles of 60° at their repective centres. A vertical pole at O subtends angle α *and* β respectively at A and B then height of the pole is

(1) $-\cot\alpha$ (2) $b\cot\beta$

(3) $\frac{a+b}{\cot\alpha+\cot\beta}$ (4) none of these

21. Three poles of height a, b, c stand on the same side of road and subtend an angle of 45° at a point on the line joining their feet. The pole of height a subtends an angle α at the foot of the pole of height b which subtends an angel β at the foot of the pole with height c if a > b > c,

then $\cot\alpha - \cot\beta =$

(1) $\frac{ac-b^2}{ab}$ (2) $\frac{bc-a^2}{ab}$

(3) $\frac{ab-c^2}{bc}$ (4) $\frac{ac-b^2}{bc}$

22. A tower stands at the foot of a hill whose inclination to the horizon is 9°; at a point 40 m up the hill the tower subtends at angle of 54°. The height of the tower is
(1) 17.56 m (2) 45.76 m
(3) 54.76 m (4) none of these

23. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. If after 10 seconds, the elevation is observed to be 30°, then the uniform speed of the aeroplane per hour is
(1) 120 km (2) 240 km
(3) $240\sqrt{3}\,km$ (4) $240/\sqrt{3}\,km$

24. The angle of elevation of a stationary clound from a point 2500 metres above a lake is 15 ° and the angle of depression of its reflection in the lake is 45°. The height of the cloud above the lake level is
(1) $2500/\sqrt{3}\,m$ (2) 2500 m
(3) $2500\sqrt{3}\,m$ (4) $5000\sqrt{3}\,m$

25. If a flagstaff 6 metres high placed on the top of a tower throws a shoadow of $2\sqrt{3}$ metres along the ground then the angle (in degrees) that the sun makes with the ground is
(1) 15° (2) 0°
(3) 60° (4) $\tan^{-1} 2\sqrt{3}$

26. A tower PQ stands at a pint P with in the triangular park ABC such that the sides *a*, *b*,*c* of the triangle subtends equal anglea at P, the foot of the tower and the tower subtends anglea α, β, γ at A, B,C respectively, then $a^2(\cot\beta - \cot\gamma) + b^2(\cot\gamma - \cot\alpha)$ $+c^2(\cot\alpha - \cot\beta)$ is equal to
(1) – 1 (2) 0
(3) 1 (4) *a* + *b* + *c*

27. A spherical balloon subtends an angle 2α at man's eye and the elevation of its centres is β. If θ is the elevation of the highest point of the balloon at A then $\tan\theta$ is equal to

(1) $\frac{\sin\alpha + \cos\beta}{\sin\beta}$ (2) $\frac{\sin\alpha + \sin\beta}{\cos\beta}$

(3) $\frac{\sin\alpha + \cos\beta}{\sin\alpha}$ (4) $\frac{\sin\alpha + \sin\beta}{\cos\alpha}$

28. Three poles whose feet lie on a circle subtend angle α, β, γ respectively at the centre of the circle. If the height of the poles are in A.P. then $\cot\alpha, \cot\beta, \cot\gamma$ in
(1) A.P. (2) G.P.
(3) H.P. (4) none of these

29. A person stands at a point A due south of a tower and observes that its elevation is 60°. He then walks westwards towards B, where the elevation is 45°. At a pint C on AB produced, he finds it to be 30°. then AB/BC is equal to
(1) 1/2 (2) 1
(3) 2 (4) 5/2

30. a pole standa at pont A on the boundary of a circular park of radius *a* and subtends an angle α at another point B on the boundary. If the chord AB subtends an angle α at the centres of the path, the height of the pole is
(1) $2a\cos(\alpha/2)\tan\alpha$
(2) $2a\sin(\alpha/2)\cot\alpha$
(3) $2a\sin(\alpha/2)\tan\alpha$
(4) $2a\cos(\alpha/2)\cot\alpha$

31. A, B, C are three points on a vertical pole whose distances from the foot of the pole are in A.P. and whose of elevation at a point on the ground are $\alpha, \beta\ and\ \gamma$ respectively. If $\alpha + \beta + \gamma = \pi$, then $\tan\alpha$ and γ is equal to
(1) 3 (2) 2
(3) 1 (4) –1

32. A, B, C are three pionts on a horizontal line through the base O of a pillar OP, such that OA, OB, OC in A.P. If α, β, γ the angles of

elevation of the top of the pillar at A, B,C respectively are also in a.P. then $\sin\alpha, \sin\beta, \sin\gamma$ are in

(1) A.P. (2) G.P.
(3) H.P. (4) none of these

33. A ladder rests against a wall at an angle of 35°. Its is pulled away through a distance *a*, so that it slides a distance *b* down the wall, finally makening an angle 25° with the horizontal, then *a*/*b* =

(1) 1 (2) $1/\sqrt{3}$
(3) $\sqrt{3}$ (4) $\sqrt{3}/2$

34. The angle of elevation of the top Q of a tower PW at a point a on the horizontal plane through P the foot of the tower is α. At a point of B on AQ at a vertical height of *a*, the angle of elevation of the middle point R of the tower PQ is β, then the height of the tower is

(1) $\frac{2a(\tan\alpha - \tan\beta)}{\tan\alpha - 2\tan\beta}$
(2) $\frac{2a(\tan\alpha - 2\tan\beta)}{\tan\alpha - \tan\beta}$
(3) $\frac{2a(\tan\alpha\tan\beta - 1)}{2\tan\alpha\cot\beta - 1}$
(4) $\frac{2a(\tan\alpha\cot\beta - 1)}{2\tan\alpha\cot\beta - 1}$

35. A lamppost stands in the centre of a circular garden and makes angle α at pint A and B on the boundary where AB subtends an angle 2β at the foot of the lamppost. If γ is the angle which the lamppost subtends at C, the middle point of the line joining a and B, then $\tan\gamma =$

(1) $\tan\alpha\tan\beta$ (2) $\sec\alpha\tan\beta$
(3) $\tan\alpha\sec\beta$ (4) none of these

36. From a pint on the ground, if the angle of elevation of a bird flying at constant speed in horizontal direction, measured at equal intervals of time are α, β, γ *and* δ

(1) $\cot^2\beta - \cot^2\gamma = 3(cot^2\alpha - \cot^2\delta)$
(2) $\cot^2\beta - \cot^2\delta = 3(cot^2\alpha - \cot^2\gamma)$
(3) $\cot^2\gamma - \cot^2\delta = 3(cot^2\alpha - \cot^2\beta)$
(4) $\cot^2\alpha - \cot^2\delta = 3(cot^2\alpha - \cot^2\gamma)$

37. A vertical tower standing at O has marks P,Q,R,S at heights of 1 m 2 m, 3 m and 4 m from the foot O and A is a point on the horizontal plane through O. If PQ and RS subtend angles α *and* β respectively at A where OA = 2m then $\cos(\alpha+\beta) =$

(1) $5/\sqrt{26}$ (2) $24/\sqrt{650}$
(3) $23/\sqrt{650}$ (4) $1/\sqrt{26}$

38. ABCD is a rectangular field with AB = *a* and BC = *b*. A lamp oost of height *h* at A subtends an angle α at P, the middle piont of CD and another lamp post of equal height at D subtends an angle β at Q, the middle point of BC. If PQ subtends an angle θ at A, then $\cot^{-1}\beta\cos^2\theta = k^2$, where k=

(1) $(a^2+b^2)2h^2$ (2) $(a^2-b^2)2h^2$
(3) $2h^2/(a^2=b^2)$ (4) $2(a^2+b^2)h^2$

39. A vertical tower OP of height *h* subtends angle α, β, γ respectively at the points A, B, C on the horizontal plane through the foot O of the tower. A is due west of the tower. B is due east of A and on the same side of the tower as A. A is due south of B, AC =

(1) $h(\cot\alpha - \cot\beta)$
(2) $h\sqrt{\cot^2\gamma - \cot^2\beta}$
(3) $h\sqrt{\cot^2\alpha + \cot^2\gamma - 2\cot\alpha\cot\beta}$
(4) $h\sqrt{\cot^2\gamma + \cot^2\beta - 2\cot\alpha\cot\beta}$

40. PQ and RS are two vertical towers of the same height. The line joining the top P and the foot S of the two towers meets the horizontal line through Q at a point A where the angles of elevation of the tops P and R of the two towers is α *and* β repectively. If AS = *a*, the height of the towers is

(1) $\frac{a\sin(\beta-\alpha)}{\cos\beta}$ (2) $\frac{a\cos(\beta-\alpha)}{\cos\beta}$
(3) $\frac{a\sin(\beta-\alpha)}{\sin\beta}$ (4) $\frac{a\cos(\beta-\alpha)}{\sin\beta}$