

QUESTION BANK OF ALGEBRA**(B.A-1, B.SC-1)****ASSINGMENT-1**

1. Prove that the NASC for a matrix A to be hermitian is that $A^\theta = A$
2. Prove that the NASC for a matrix A to be skew hermitian is that $A^\theta = -A$
3. Prove that every matrix can be expressed in one and only one way as $P + iQ$ where P and Q are hermitian.
4. Show that every hermitian matrix A can be uniquely expressed as $P+iQ$ where P and Q are real symm. And real skew symm. Also show that $A^\theta A$ is real iff $PQ = -QP$
5. Define unitary matrix , orthogonal matrix
6. Define Rank of a matrix
7. Find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$
8. Define elementary matrix .
9. Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 3 \end{bmatrix}$ as a product of elementary matrices.

10. Find rank of the matrix:
$$\begin{bmatrix} 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -3 & 1 \\ 0 & 1 & 1 & 2 & -6 \\ 3 & -4 & -7 & -8 & -2 \end{bmatrix}$$

11. Define normal form of a matrix.

12. Reduce the matrix
$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$
 to the normal form.

13. Let $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ find non singular matrices P and Q such that PAQ is in normal form

14. Find the inverse of
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

15. Define row and column rank of a matrix.

16. Reduce to row echelon form the matrix $A = \begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$

17. Write the vector $V = (3, 2, 1)$ as a linear combination of the vectors

$$v_1 = (2, -1, 0), v_2 = (1, 2, 1), v_3 = (0, 2, -1)$$

18. Is the vector $V = (2, -5, 3)$ a linear combination of the vectors

$$v_1 = (1, -3, 2), v_2 = (2, -4, -1), v_3 = (1, -5, 7) ?$$

19. Under what condition on scalar a are the vectors $(1-a, 1+a)$ and $(1+a, 1-a)$ L.I.

20. Prove that vectors $x = (3, 0, -3)$; $y = (-1, 1, 2)$; $z = (4, 2, -2)$; $t = (2, 1, 1)$ are L.D.