SELF ASSESSMENT TEST -2

B.A-2

Advanced calculus

(Limit & Continuity of two variables and Partial Derivatives)

1. By using definition ,prove that $\lim_{\substack{x \to 0 \\ y \to 0}} \left(x^2 + y^2\right) = 0$

2. Let f (x,y) = y sin $\frac{1}{x}$ + x sin $\frac{1}{y}$,where x $\neq$ 0, y$\neq$ 0 prove that

   f(x,y) $\to$ 0 as (x,y) $\to$ 0.

3. Prove that $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$

4. Let f : $R^2 \to$ R be define by f (x,y) = $\begin{cases} 1 & x \in R-Q \\ 0 & x \in Q \end{cases}$

   Show that for any pt. (x,y) $\to$ (a,b) , f (x,y) does not exist

5.Prove that $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist.

6. Prove that $\lim_{(x,y)\to(0,0)} (x+y)\left\{\frac{y+(x+y)^2}{y-(x+y)^2}\right\}$ does not exist.

7.Let f(x,y) = $\begin{cases} \frac{x^2 y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Prove that a st. line approach gives the limit zero, but the limit does not exist.

8.Prove that $\underset{(x,y)\to(0,0)}{Lt}\dfrac{x^3}{x^3+y^3-x}$ does not exist

9.using defination discuss the cont. f(x,y) = $\begin{cases} xy\sin\dfrac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$ at origin

10.Let f: $R^3\to$ R be define by f(x,y,z)= $x^2+3y^2+5z^2$ show that f is cont.

11.Let f : $R^2\to$ R be cont. Define g : $R^2\to$ R as

g(x,y) = $\begin{cases} f(x,y) & (x,y)\neq(0,0) \\ f(x,y)+1 & (x,y)=(0,0) \end{cases}$ .show that g is not cont. at (0,0)

12.Show that the function f(x,y) = $|x|+|y|$ is cont. at the origin.

13.Show that the function f(x,y)= $\begin{cases} \dfrac{xy^3}{x^2+y^6}, x\neq 0, y\neq 0 \end{cases}$ and f(0,0)=0

Is not cont at (0,0) in (x,y) together but that function is cont in x alone and in y alone at the origin.

14.Let f: $R^2\to$ R define by f (x,y) = $\begin{cases} 1 & x\in R-Q \\ 0 & x\in Q \end{cases}$

Show that f is not continuous at any point of R^2

15.f: $R^3\to$ R be cont. function. Define g : $R^3\to$ R as

g(x,y,z)= $\begin{cases} f(x,y,z) & (x,y.z)\neq(0,0,0) \\ f(x,y,z)+1 & (x,y,z)=(0,0,0) \end{cases}$ prove that g (x,y,z) is not cont.at (0,0,0)

16.Let f,g:R$\rightarrow$ R be two cont. function. Let F : $R^2 \rightarrow$ R be defined by F(x,y)=f(x)+f(y). Prove that F is a cont. function.

Partial Derivatives

17.If $\theta = t^n \cdot e^{\frac{-r^2}{4t}}$, find the value of n which will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$.

18. If u =log($x^3 + y^3 + z^3 - 3xyz$) , show that

(a) $\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)^2 = \frac{9}{(x+y+z)^2}$.

(b) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$

19.If u = f(r) where $r^2 = x^2 + y^2$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = f''(r)+\frac{1}{r}f'(r)$.

20.If v = $\left(x^2+y^2+z^2\right)^m$, show that

$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2} = 2m(2m+1)\left(x^2+y^2+z^2\right)^{m-1}$.

21.Let f(x ,y) = xy$\left(\frac{x^2-y^2}{x^2+y^2}\right)$ where (x ,Y) $\neq$ (0,0) and f(0,0)=0 show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

22.Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$ for a function f(x,y) define as f(x,y) = $\begin{cases} x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right) & if \quad xy\neq 0 \\ 0 & if \quad xy=0 \end{cases}$.

23.State and prove Euler's theorem of homogeneous functions of two variable.

24.State and prove Euler's thm on homogeneous functions of three variable.

25.Verify Euler's thm for z= $\frac{x^4+y^4}{x-y}$.

26.If z= $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{1}{2}\tan z$.

27.If z= $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=\left(1-4\sin^2 z\right)\sin 2z \ .$$

28.show that the f(x,y)= sinx + cosy is diff. at every pt of R^2.

29.Show that the f(x,y)= cos(x+y) is diff. at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$.

30. Show that the function f(x,y) = $|x|+|y|$ is cont. at (0,0) but not diff. at (0,0).

31.Show that f(x,y) = x y^2 is diff. at (1,2)

32. State and prove Schwarz's thm.

33. State and prove Young's thm.

34.If H = f(y-z,z-x,x-y), prove that $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$.

35.If z= f(u,v) where u= e^{x}cosy , v = e^{x} siny , show that

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\left(u^2+v^2\right)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$$.

36.State and prove Taylor's thm for function of two variable .

37.Prove that e^{ax} sinby = by + abxy + $\frac{e^{atx}}{b}$ [($a^3x^3-3ab^2xy^2$) sinbty +

($3a^2bx^2y-b^3y^3$) cosbty] where 0<t<1.

38.Use Taylor's thm to expand xy^2 + 3x -2 in process of x+2 and y-1.