SELF ASSESSMENT TEST -4
CLASS 10+1
COMPLEX NUMBERS

1.Evaluate (a) $i^{37}+\frac{1}{i^{67}}$ (b) $i^{100}+i^{101}+i^{102}+i^{103}$ (c) $1+i^{2}+i^{4}+.............+i^{20}$ .ans 2i,0,1

2.Evaluate (a) $\left[i^{19}+\left(\frac{1}{i}\right)^{25}\right]^{2}$ (b) $\sum_{n=1}^{100} i^{n}$ (c) $(1+i)^{4}\left(1-\frac{1}{i}\right)^{4}$ .ans. -4,0,16

Note. For any two real numbers $\sqrt{a}\sqrt{b}=\sqrt{ab}$ is true only when at least one of a and b is non negative. In other words is $\sqrt{a}\sqrt{b}=\sqrt{ab}$ not valid if a and b both are negative.

3.Evaluate (a) $\sqrt{-25}\sqrt{-49}$ (b) $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$ ans -35,17i

4.Find the value of x and y (a) (2x+y-5)+(3x-2y-4)i=0 (b) $(1+i)y^{2}+(6+i)=(2+i)x$

(c) $(x^{4}+2xi)-(3x^{2}+iy)=(3-5i)+(1+2iy)$. ans (a) x=2,y=1,(b) x=5,y=$\pm$ 2

5.simplify (a) $(1-i)^{4}$ (b) $\frac{1}{(1-i)^{2}}-\frac{1}{(1+i)^{2}}$ (c) $(\sqrt{5}+7i)(\sqrt{5}-7i)^{2}$ (d) $\left(-2-\frac{i}{3}\right)^{3}$.

ans.(a) -4, (b) i (c) 54$\sqrt{5}$ -378i (d) $\frac{-22}{3}-\frac{107i}{27}$.

6.Express in the form of x+iy. (a) $\frac{(2+i)^{3}}{3+2i}$ (b) $\frac{(2+3i)(3+2i)}{5+i}$ (c) $\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$.

Ans. (a) $\frac{28}{13}+\frac{29}{13}i$ (b) $\frac{1}{13}-\frac{5i}{13}$ (c) $\frac{307}{442}+\frac{599i}{442}$.

7.If z_1, z_2 are 1-i , -2+4i resp. ,find $\mathrm{Im}\left(\frac{z_1 z_2}{\bar{z}_3}\right)$. ans 2

8.If z= -5 +4i ,show that $z^2+10z+41=0$ and hence find the vaule

$z^4+9z^3+35z^2-z+4$. ans. -160

9.Find θ s.t. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. Ans. Sinθ =0

10.If $a+ib=\frac{c+i}{c-i}$; show that $a^2+b^2=1$ and $\frac{b}{a}=\frac{2c}{c^2-1}$.

11.Given that $(1-5i)z_1-2z_2=3-7i$ find z_1 and z_2 where z_1 & z_2 are conjugate of each other. Ans. z_1 =2+I, z_2 =2-i

12.If $|z+i|=|z-i|$,find z . ans. z=x

13.if $|a+ib|=1$,then show that $\frac{1+b+ai}{1+b-ai}=b+ai$.

14.Find the number of non zero integral solutions $|1-i|^x=2^x$. ans.0

15.If $(1+i)(1+2i)........(1+ni)=x+iy$,show that $2.5.10.........(1+n^2)=x^2+y^2$.

16.Find the modulus of $\frac{(1+i)^{2n+1}}{(1-i)^{2n-1}}$. ans.2

17.Let z= x+iy and $w=\frac{1-iz}{z-i}$ .If $|w|=1$, then show that z is purely real.

18.if z is complex number s.t. $\frac{z-1}{z+1}$ is purely img. Show that $|z|=1$.

19.If z=x+iy prove that $|x|+|y|\leq\sqrt{2}|z|$.

20.If $|z_1|=|z_2|=...........=|z_n|=1$,then show that

$$|z_1+z_2+.........+z_n|=\left|\frac{1}{z_1}+\frac{1}{z_2}+..........+\frac{1}{z_n}\right|$$

21.If z = x + i y and $z\bar{z}-(2+3i)z-(2-3i)\bar{z}+9=0$, show that $(x-2)^2+(y+3)^2=4$.

22.If z =x+iy and $\left|\frac{z-5i}{z+5i}\right|=1$,show that z is purely real.

23.Find the square root of the 1-4$\sqrt{-3}$.

24.If α & β are the two cube roots of unity , prove that $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=0$.

25.Solve the equation $(x-2)^3+27=0$.

26.write $\left(i-\sqrt{3}\right)^{13}$ in algebraic form.

27.Write $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ in polar form and their modulus and argument .