TEST -3
CLASS B.A-3/B.SC-3

NUMBER THEORY

1. For any Positive integer m, prove that (ma,mb) = m(a,b).

2. Prove that if m > n then $a^{2^n}+1$ is a divisor of $a^{2^m}-1$. Also show that if m,n are positive integers with m $\neq$ n then $\left(a^{2^n}+1, a^{2^m}-1\right)=\begin{cases}1, & a \quad even\\ 2, & a \quad odd\end{cases}$.

3. Find gcd of 710 and 68 and find integers x and y satisfying (710,68)=710x+68y

4. Verify that $2^{2^5}+1$ is divisible by 641.

5. Find the last two digits of numbers 9^{9^9} and 2^{1000}.

6. For any positive integer n > 1, prove that $\phi(n)=n\prod_{p|n}\left(1-\frac{1}{p}\right)$.

7. State and prove Euler Fermat thm.

8. Solve x $\equiv$1(mod 4), x $\equiv$0 (mod 3), x $\equiv$ 5 (mod 7)

9. State and prove Wilson's thm.

10. Using Wilson's thm ,Show that 17 is a prime.