

TEST SERIES -1
CLASS B.A-2/B.SC-2
(ANALYSIS)

Section-A

1(a) If f be bdd function defined on $[a,b]$, then $\int_{-a}^b f dx \leq \int_a^{-b} f dx$. (4)

(b) Show that the function defined by $f(x) = \begin{cases} 2, & x \in Q \\ 1, & x \in Q' \end{cases}$ is not integrable on any interval. (3 $\frac{1}{2}$)

2(a) Proceeding From the defination compute the integral $\int_a^b x^m dx$. (5)

(b) If f is a non negative cont function on $[a,b]$ and $\int_a^b f(x)dx = 0$ prove that $f(x)=0$ (

2 $\frac{1}{2}$)

3(a) If f is R-integrable on $[a,b]$ and F is a function defined on $[a,b]$ s.t. $F'(x) = f(x)$ then.

$\int_a^b f(x)dx = F(b) - F(a)$ (4)

(b) If $0 < a < b$, show that $\left| \int_a^b \frac{\sin x}{x} dx \right| \leq \frac{2}{a}$. (

3 $\frac{1}{2}$)

4. If a function f defined on $[a,b]$ is bdd and has a finite number of pts of discontinuity , then f is R.I. (

7 $\frac{1}{2}$)

Section-B

5(a) Prove that the seq $\left\{ \left(1 + \frac{3}{n} \right)^n \right\}$ is monotonically increasing and bdd. Prove that it cgt to

the limit e^3 .

(4)

(b) Prove that the series $\sum_{n=2}^{\infty} a_n$ where $a_n = \frac{1}{n^p (\log n)^q}$ cgs if either $p > 1$ or $p = 1$ and $q > 1$.

(3)

(6) Prove that NASC for the the cgs of a seq of a real nos is that it is a cauchy seq.

(7)

(7)(a) Discuss the cgs of series $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$, $x > 0$.

(4)

(b) Show that the seq $\{a_n\}$ where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ does not cgt, by showing that it is not cauchy seq.

(3)

8(a) State and prove cauchy's Root test.

(4)

(b) Test the series $1 + \frac{\alpha\beta}{1\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2\gamma(\gamma+1)}x^2 + \dots$ for cgs

(3)

Section –c

9(a) Define Partition, refinement, lower R.I, upper R.I.

(b) Show that $\int_0^1 [5x] dx = 2$.

(c) Prove that $\int_0^1 x^2 dx \leq \int_0^1 \sqrt{x} dx$.

(d) Prove that seq $\left\{ \left(1 + \frac{1}{n} \right)^n \right\}$ is bdd

(e) show that $\lim_{n \rightarrow \infty} \left[\frac{(n+1)(n+2)\dots(n+n)}{n^n} \right]^{\frac{1}{n}} = \frac{4}{e}$.

(f) If $\sum_1^{\infty} a_n$ is cgt, then prove that $\lim_{n \rightarrow \infty} a_n = 0$.

(g) Discuss the cgs $\sum (-1)^{n-1} \frac{n+1}{n}$.

(h) Define Ratio test.

(i) Define Gauss's test (j) Define logarithm's test

10 x 1 = 10

Goyal's Math