

TEST -2**CLASS B.A-1/B.SC-1****GEOMETRY**

Section -A

1(a). Find the angle through which the axes may be rotated so that the eqn

$$17x^2 + 12xy + 8y^2 + 13x - 17y + 20 = 0 \text{ may be wanting in the mixed term } xy. \quad (4)$$

(b). Show that the lines joining origin to the pts of intersection of $x^2 + y^2 + 2gx + c = 0$ and

$$x^2 + y^2 + 2fx - c = 0 \text{ are at right angles if } g^2 - f^2 = 2c. \quad \left(3\frac{1}{2}\right)$$

2(a). Prove that the locus of the poles of chords which are normal the parabola $y^2 = 4ax$ is the

$$\text{curve } y^2(x+2a) + 4a^2 = 0. \quad (4)$$

(b). A tangent to the parabola $y^2 + 4bx = 0$ meet the parabola $y^2 = 4ax$ in P and Q. Show that

$$\text{the locus of the middle pt of PQ is } y^2(2a+b) = 4a^2x. \quad \left(3\frac{1}{2}\right)$$

3(a). Prove that the semi latus rectum of a parabola is the harmonic mean between the segment of a focal chord. (4)

(b) Show that tangent at any pt P of the parabola bisects between focal chord to P and

$$\text{perpendicular from P to directrix.} \quad \left(3\frac{1}{2}\right)$$

4. Prove that eqn $x^2 + 2xy + y^2 - 2x - 1 = 0$ represents a parabola and find its focus, latus rectum and directrix. Also trace the curve.

Section-B

5(a). Show that the chord of contact of perpendicular tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{always touch the ellipse } \frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2 + b^2}. \quad (4)$$

(b). If the normal at the end of latus rectum of an ellipse passes thro. one extremity of the minor axis. Show that the eccentricity of the curve is given by the eqn $e^4 + e^2 - 1 = 0$. $(3\frac{1}{2})$

6(a). Prove that the locus of the middle pts of the portion of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ included between axes is the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$.

(b). If P and D are the extremities of conjugate diameters on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the tangents at P and D meet on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ and that the locus of the middle point of PD is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$.

7(a). Find asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ and find general eqn of all hyperbola having the same asymptotes.

(b) Tangents are drawn from a pt $(-2, -1)$ to the hyperbola $2x^2 - 3y^2 = 6$. Find their eqns and angle between them

8. If a diameter meets one hyperbola in real pts then conjugate diameter will meet the conjugate hyperbola also in real pts.

SECTION-C

9(a). Define Rectangular hyperbola, Conjugate hyperbola.

(b). Find the pole of line $x - 2y = 1$ w.r.t. the hyperbola $4x^2 - 9y^2 = 12$.

(c) Find the eccentricity of ellipse $x = 3 \cos A$, $y = 2 \sin A$.

(d) Define eccentric angle and auxiliary circle.

(e) Find whether the pts $(0, 1), (-1, 2)$ lie in interior or exterior of the $4x^2 + 9y^2 = 36$. $5 \times 2 = 10$