TEST - 1
CLASS B.A-1/B.SC-1
CALCULUS

Selecting two questions from each section A and B and compulsory question of section C.

Section – A

1(a) If y = $\left(\sin^{-1}x\right)^2$ find $y_n(0)$. (4)

(b) Find all asymptotes of the $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$. ($3\frac{1}{2}$)

2(a) Show that curve x = $\log\left(\frac{y}{x}\right)$ has a pt of inflection at $\left(-2,-2e^{-2}\right)$. ($3\frac{1}{2}$)

(b) Show that pts of intersection of the curve $xy\left(x^2-y^2\right)-25x^2-9y^2+144=0$ and its asymptotes lie an ellipse whose eccentricity is 4/5 (4)

3. Trace the curve $x\left(x^2+y^2\right)=a\left(x^2-y^2\right)$. a > 0 ($7\frac{1}{2}$)

4. Find the center of curvature at any pt (x,y) of the parabola $y^2=4ax$ .Also find evolute. ($7\frac{1}{2}$)

Section – B

5(a) If $I_n=\int_0^{\frac{\pi}{2}}x^n\sin x dx$ and n> 1 prove $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$. (4)

(b). Find the length of the arc of the parabola $y^2-4y+2x=0$ which lies in the 1st quadrant.

6 (a) Find the total area of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$. ($3\frac{1}{2}$)

(b) Show that $\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x}$ is cgt. (4)

7(a) If g bdd and monotonic and tends to 0 and $\int_a^t f dx$ is bdd for t>a then $\int_a^\infty fg dx$ cgt at ∞ .(4)

(b) Show $\int_0^{\frac{\pi}{2}} x^m \operatorname{cosec}^n x dx$ exist iff n < m+1. $(3\frac{1}{2})$

8. If $|a|<1$ evaluate $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x}dx$. $(7\frac{1}{2})$

Section-C

9(a) If y = cosh(logx) +sinh(logx) prove y_2 =0.

(b) Define concave upward, downward and pt of inflection.

(c) Define double pt ,node ,cusp ,conjugate pt.

(d) Show that curvature of a st line is zero.

(e) Show that parabola $y^2 = 4ax$ has no asymptotes.

(f) Integrate $\int \frac{dx}{3\cosh x + 5\sinh x}$.

(g) Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}dx$.

(h) Evaluate $\int_0^1 x^5(1-x^3)^3 dx$.

(I) show $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$.

(j) a>0, b>0 prove $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}dx = \log\frac{b}{a}$. 10 x 1 =10